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The demand for financial assets: a portfolio approach

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CHAPTER I. INTRODUCTION AND HISTORICAL BACKGROUND

Introduction and Purpose

The purpose of this study is to determine the demand for various assets which compose the net wealth of the private sector. This chapter provides a detailed statement of the problem to be studied and a historical context within which this problem may be viewed. Chapter II contains a discussion of the relevancy of the own rates of return to the individual demand functions. In addition, a portfolio model derived from these considerations is presented and two alternative specifications of the model are discussed. Provided in Chapter III are the empirical results obtained by estimating the demand functions under both specifications as well as results obtained by relaxing various assumptions of the model. Finally, Chapter IV contains a summary and conclusions of this study.

Historical Background

In a general equilibrium framework, any economic variable may be recognized as having a direct or indirect affect upon any other economic variable. Two important simultaneous developments in recent monetary economics which aid in assessing these effects are the role of wealth in spending behavior and portfolio analysis. The former was a response to the theoretical specification of the linkage between monetary and value theory. An outgrowth of increased emphasis upon general equilibrium analysis and econometric model-building, the latter appears to have been accepted by both major "schools" of monetary theory.

Wealth Effects

An investigation of the development of the theory expounding the direct affect of wealth upon spending behavior leads to the literature concerning the neutrality of money and dichotomization of the "real" sector and financial sector. This procedure is inevitable due to the synonymous usage by early writers of the terms real value of money and wealth. Hence, the issues concerning the "veil of money" and the separability of the "real" sector and financial sector, i.e., the independent determination of the "real" variables in the "real" sector, simultaneously became an issue of the assessment of direct wealth effects induced by changing monetary policy. For the purposes of this study, a detailed description of the investigations surrounding both issues is not necessary. An adequate summary of these investigations may be found in Johnson (1962). Any summary of these debates would most likely contain three stages; these are: (1) "Pigou effect;" (2) "real balance effect;" and (3) "outside" versus "inside" money.

After Keynes' (1936) attack upon the classical system of automatic full-employment with flexible prices, several authors attempted to rebuff his arguments by providing a mechanism consistent with classical theory which integrated monetary and value theory. In other words, they sought a mechanism through which money affected relative prices as well as the absolute price level and which left the basic classical system intact. Among the more important works of the early defenders are those of Pigou (1943), Scitovszky (1940), and Haberler (1941). An outgrowth of their work was the use of the "Pigou effect" to restore the internal consistency of the classical system. The "Pigou effect" is the resulting behavior of the addition of a liquid wealth asset to the saving function. It was contended that this additional variable and the implied mechanism was the answer to creating an integrated macroeconomic model. Although these early investigations did not directly address themselves to the role of wealth in influencing economic behavior (they spoke instead of the role of the real value of money), these authors had introduced wealth into the system in a new way. Wealth became the link between value and monetary theory and thus earned more important consideration within the theory of spending behavior.

When the neutrality debate continued much later, Metzler (1951) argued that these early attempts may have succeeded at creating non-dichotomization at the expense of still another important feature of the classical system. The inclusion of the "Pigou effect" in order to attack the problem of the inconsistency of saving and investment produced a new breed of interest theory. Metzler claimed that the Pigou-Scitovszky-Haberler formulations led to a theory of interest which was unlike the polar extremes of either the Classicals or Keynes. This new version depended upon the way in which changes in the quantity of money occurred. If the change occurred through open-market operations, then a "monetary" theory of interest resulted; however, if the change occurred without any offsetting changes in private holdings of other assets, then a "real" theory of interest resulted, i.e., non-neutral and neutral monetary theory, respectively. It should be noted that the direct affect of wealth, as subsequent writers pointed out and accepted, in Metzler's open-market case depends upon a distributional affect which is generally assumed negligible in most economic analysis.

Two monumental scholarly works followed these investigations; these are the works of Patinkin (1954) concerning the role of "real balances" and Gurley and Shaw (1960) concerning the role of financial intermediaries. Patinkin's attempts to integrate monetary and value theory were centered upon the explicit introduction of "real balances," i.e., the real quantity of money, into the demand for commodities. It was his contention that classical theory could be reconstituted by noting the role "real balances" played. Much of his work is devoted to systematic model building in which he continually notes the conditions necessary for a neutral monetary theory. Patinkin extended his analysis beyond the commodity market and encompassed both the money and bond markets. His extensions continued the general theme of noting the role of "real balances."

Gurley and Shaw extended the analysis of neutrality to include other types of financial instruments which they viewed as close substitutes for money, e.g., savings and loans deposits. In attempting to analyze assets of varying liquidity, a new technique of viewing the liquid wealth component included in the demand for various assets was developed. Gurley and Shaw spoke of what they termed "outside" and "inside" money and used these distinctions to assess the neutrality of monetary policy. "Outside" money was defined as government debt which is not directly or indirectly the liability of another individual within the sector being considered. On the other hand, "inside" money was defined as government debt which simultaneously represents the liability of an individual internal to the sector. They concluded that neutrality existed only in the case where wealth was defined as "inside" money and nonneutral in several other cases.

The debate concerning the direct affect of wealth upon spending behavior would appear to have resolved to one concerning the definition of wealth itself. This is one of the important issues to which Warren Smith (1970) addresses himself in an attempt to survey current issues in monetary economics. Smith claims that analysis of the effect of wealth in the portfolio adjustments of the private sector should contain as net wealth of the sector only those assets which are not liabilities of individuals within the sector. Defining "outside" money as claims against the central government which take the form of demand debt, i.e., currency and deposit claims against the central bank, he considers "outside" money an obvious inclusion within the net wealth of the public sector. In his context, the public sector in a closed economy includes households, businesses, and the banking system. Metzler's issue concerning the wealth effects of open-market operations hinges upon the way the public sector views government debt which is not demand debt, e.g., interest-bearing debt of various maturities. That is, the neutrality of the monetary sector hinges upon whether this sector views the latter form of government debt as an asset with no concomitant liability. Monetary disturbances will be neutral in the sense that monetary policy has no direct wealth effects, i.e., ability to create and destroy wealth through changes in "outside" money, if interest-bearing government debt is viewed as both an asset and liability.

If this form of debt represents only an asset then a non-neutral theory is implied and changes in open-market operations have a direct wealth affect as well as any induced portfolio adjustment affect. Smith concludes that both forms of government debt should be included within the definition of net wealth and that open-market operations do not affect the nominal wealth holdings of the sector. While Smith is willing to accept the inability of open-market operations to bring about direct wealth effects through changing nominal wealth, he warns that monetary policy may bring about important wealth effects via interest rate changes. Declining interest rates, induced by expansionary monetary policy, represent declining capitalization rates applied to future income streams, hence, raising the real value of wealth holdings. Therefore, Smith concludes that monetary policy may have direct wealth effects. Since "inside" money, i.e., claims against the private sector, does not meet his requirement for inclusion within net wealth, no further considerations must be made with reference to this asset.

Portfolio Analysis

The affect of wealth upon economic activity may be direct as discussed above, but the inclusion of wealth as an important determinant of spending behavior requires that the indirect affect of wealth also be assessed. Portfolio analysis may be used to assess the effect of the wealth adjustment process. The most striking similarities of investigations concerning either the demand for money or the determination of the relative prices of various assets occur due to the usage of portfolio analysis. Unfortunately, all too often, the term portfolio analysis has been used to distinguish between the Keynesian and neo-classical theorists. Since Keynes introduced bonds as an alternative to holding money, the analysis of the demand for money has continued to assess the affect of various asset holdings upon this demand via portfolio adjustments. Hence, the Keynesian followers have identified themselves with portfolio analysis; however, neo-classical economists or monetarists engage in portfolio analysis also.

While the usage of this technique of analysis produces many similarities in these alternative "schools" of thought, the theoretical implications differ as may be illustrated by reviewing the works of the major authors of each approach, James Tobin and Milton Friedman, respectively. Keynesian monetary theory expounded by Tobin (1965) and earlier by James Duesenberry (1963) is an application of general equilibrium theory to the determination of the relative prices (yields) of various assets. Within this framework, both the process of accumulating wealth and the form in which wealth is held must be investigated. The separation of decisions involving accumulation and allocation of wealth is a key behavioral assumption of this approach. Accumulating wealth, i.e., saving, is viewed as primarily determined by income. Allocation of wealth, assuming some form of optimizing behavior, is a function of the various yields and risks associated with the available assets. This theoretical approach to portfolio analysis provides no "special" place for money within equilibrium analysis. Emphasis is placed upon the substitution effects of money as an alternative form of holding wealth. Within the Keynesian framework, the demand for money is part of consumer theory.

On the other hand, monetarists view money as part of capital theory and by emphasizing the income effect associated with the demand for money, these economists have afforded money the central position in analysis of economic activity. Milton Friedman is the primary author of the monetarists' approach and has laid the theoretical foundation for this approach in his 1956 article and reiterated this approach in a recent article (Friedman 1970). Friedman uses the quantity theory of money as his starting point and moves from a transactions version which emphasizes money as a means of payment to an income version which emphasizes money as a temporary abode of purchasing power. He argues that viewing money in the latter version leads to stressing the desire to hold money as an asset and, hence, its comparative usefulness as an asset, i.e., portfolio analysis. Viewing wealth, in the broadest sense, as including all sources generating income or income-in-kind, Friedman treats the demand for money as part of capital theory. Thus, the demand for money by an ultimate wealth holder is viewed in the same manner as the demand for any consumption service. The demand for any asset becomes a function of three factors: (1) wealth constraint; (2) the relative prices (yields) of alternative assets; and (3) preferences of those holding wealth. Friedman's approach varies from that of the Keynesians by including human capital within the definition of wealth. Ultimately, his concern for the comparative relevance of wealth and current income leads Friedman to another innovation, i.e., permanent income. The monetarists position can be summarized by noting: (1) the role interest rates play [in particular, the interest-inelastic demand for money]; (2) the importance of permanent income; and (3) the broadened definition of wealth. Although the monetarists designate money as the prime mover of economic activity and focus attention upon this single asset, they engage in portfolio analysis and this technique has produced many qualitative similarities in empirical analyses.

Recently, Tobin has published two articles (Brainard and Tobin, 1968, and Tobin, 1969) in which he outlines a general equilibrium approach to monetary economics as well as warns against certain pitfalls encountered in financial model-building leading to qualitative errors of specification. The major contribution of these recent articles is the emphasis placed upon the explicit recognition of the interrelatedness of the markets for various financial assets. Tobin argues that once the array of assets held by a sector is determined, the entire list of interest rates relevant to these assets must appear in the demand function of each asset. Further, should the cross-effects of these rates appear empirically insignificant, i.e., should a rate of return appear insignificant in a demand equation other than its own, there is no reason to omit any rate since the sum of the changes in demand relative to the change in a particular rate is not zero but rather equal in absolute terms to that of the own-effect. In other words, the total effect of a particular interest rate change, summed over all assets in the portfolio, must be zero. Also, he notes that the inclusion of a rate of return in only one of the demand functions implicitly assumes that movements in that asset occur at the expense of a residual asset whose demand function remains unspecified.

In reviewing the theoretical background concerning portfolio analysis of the financial sector, it seems clear that Warren Smith and James Tobin have provided a framework in which a model of the public sector may be specified and empirically tested. This is the problem which this study proposes to attack.

Empirical Investigations

Before specifying a particular model, it would appear helpful to review empirical research which has taken place in this area even though these attempts may have failed to meet either Smith's or Tobin's requirements. An attempt to survey all of the empirical evidence concerning the demand for money and the determination of the relative prices of various assets is completely outside the scope of this study. Two brief attempts of such a survey are contained in a book by David Laidler (1969) and in an article published on behalf of the Bank of England (Goodhart and Crockett 1970). For the purposes of this study three representative studies shall be reviewed: these are the articles by Henry A. Latané (1954), Brunner and Meltzer (1964), and Michael J. Hamburger (1966). A summary of these works is presented in Appendix A. It should be noted that none of these empirical works presents a model of the Smith-Tobin variety upon which their empirical investigations hinge. Indeed Brunner and Meltzer recognize the need for such a model by stating, "A more fundamental issue bears on the formulation of higher level hypothesis explaining the characteristic occurence of money in asset portfolios." However, the general approach is to specify various forms of the demand for money which the authors feel represent alternative formulations based upon the two major schools of monetary economics.

H. A. Latané attempted to determine the relationship between the proportion of nominal income held in cash balances and the interest rate. Estimating linear equations (using both of the possible causal orderings) by means of ordinary least squares, Latané concludes that a joint estimating equation which is linear indicates that the proportion of income held in cash balances is interest-inelastic, -.8. Latané's early work is representative of attempts to associate the velocity of money with a rate of interest.

Brunner and Meltzer represent those authors who attempt an "exhaustive" approach, i.e., they attempt to estimate several forms of demand for money equations which purport to represent the two "schools" of thought. Three different specifications which are presented include an interest rate, real non-human wealth, and output prices. In addition two specifications include a proxy for the index of human to non-human wealth. The remaining specification contains a measure of uncertainty concerning credit market conditions. Each demand formulation was presented as a linearized approximation of a logarithmic money demand equation in order that it might be jointly estimated with a linear supply of money function. The results from the one-stage and two-stage least squares estimates reaffirm the interest-inelastic demand for money. It is of interest to note the wealth elasticity of the demand for money appears to be quite high under all specifications.

Hamburger moved a step closer to a Smith-Tobin approach by explicitly introducing the relationship between money and other financial assets into the demand for money. Hamburger attempted to resolve five issues concerning the demand for money: (1) Do liquid assets correspond more closely to the theoretical concept of money than other assets?; (2) What rates of return represent the opportunity costs of holding money?; (3) Which is a more important constraint income or wealth?; (4) How quickly do households adjust their money balances to changes in returns to assets?; and (5) Is the rate of adjustment the same for all independent variables?. Several formulations of the demand for money were estimated assuming that the function was linear in the logarithms. In order to consider the adjustment process, the response of the household sector to changes in returns to assets in their portfolio was assumed to take the form of a distributed lag. Estimation of the parameters of the model were obtained by adopting a technique developed by Nerlove for estimating the adaptive expectations hypothesis. Due to multi-collinearity, Hamburger expressed all formulations as first-difference relationships. His empirical results, once again, confirm the interest-inelasticity of the demand for money. Also, both the yield on corporate bonds and equities play an important role in the determination of the demand for money. However, Hamburger's attempts to assess the importance of income and wealth as constraints resulted in finding that neither variable was statistically significant when added as an explanatory variable.

Combined with Appendix A, this brief account of empirical results represents a large volume of empirical investigations concerning the nature of the demand for money. These varied approaches suggest that a systematic attempt to follow a Smith-Tobin model might provide many insights into the problems encountered in financial model-building.

CHAPTER II. THE MODEL

The model developed in this chapter reflects a Smith-Tobin portfolio approach to the demand for financial assets. In particular, the model constructed in this chapter is a portfolio analysis of the net wealth of the private sector. It should be noted that the term private sector signifies that this model explores the demand for financial assets on behalf of households and businesses within a closed economy. Both Smith and Tobin developed models of the public sector, i.e., their models included the banking system as "individuals" within the sector. It would appear more insight could be gained into the effects of monetary policy if the banking sector was viewed separately, hence, this model concentrates upon the private sector. Also, it should be noted that the term financial assets is used to signify that the demand for capital as explained below is not estimated. In accordance with Smith, the net wealth of the private sector is defined to include only those non-human assets of the sector which do not represent concomitant liabilities to "individuals" within the sector. Once the portfolio of assets has been investigated, the demands for the various assets are stated in alternative forms which help illuminate the complexities involved in empirical investigations.

Assuming the private sector exhibits some form of optimizing behavior, the demand for a particular asset within its portfolio depends upon the net non-human wealth which the sector possesses and the returns to the various assets.[1] Net non-human wealth of the private sector is the summation of the various assets held by this sector; assume these are: (1) money; (2) time deposits; (3) bonds; and (4) capital. Money is defined as the summation of currency and demand deposits in the hands of the non-banking public. Hence, any measure of net non-human wealth must take into consideration the concomitant liability of loans from the banking sector that may have been used to add to this and/or some other asset held by the private sector. Time deposits include only those in the hands of the non-banking public, and bonds include all forms of U. S. government debt held by the private sector. Capital is the stock of non-human assets other than the financial assets already mentioned; within this context, the purchase of shares of stock is viewed as an intermediate step in the acquisition of capital. Net

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[1]A complete statement of the optimizing behavior of the private sector would require both an objective function and the constraints upon achieving this objective. While this study has chosen to avoid these problems in order to reduce the complexity of analyses and to provide additional clarification to the behavior of more important variables commonly investigated in financial models, nevertheless, it would appear instructive to conjecture as to the nature of these functions. As suggested by Tobin, the objective function might be stated in terms of the total return from these assets, e.g.,

$$TR = \sum_{i=1}^{n} RR_i A_i$$

where, TR = total return
RR_i = real rate of return to i^{th} asset
A_i = real holdings of i^{th} asset

and the constraint in terms of the risk involved in holding a particular asset. Obviously, if the various assets did not possess different risks, individuals could maximize returns by holding only the asset with the highest return. One way to express this constraint would be in terms of the variance of the yields upon the various assets. Another constraint would be the net wealth of the sector. A more detailed description of portfolio optimization and the demand functions which can be derived from such analysis is provided in Appendix D.

non-human wealth of the private sector may be stated as follows:

$$W = M + T + (P_b \cdot B_p) + (P_k \cdot K) - L$$

where,

W = nominal net non-human wealth of the private sector

M = nominal currency plus demand deposits held by the non-banking public

T = nominal time deposits held by the non-banking public

P_b = average market price of a U. S. government bond purchased by the private sector

B_p = number of U. S. government bonds held by the private sector

P_k = average market price of a unit of capital held by the private sector

K = number of units of capital held by the private sector

L = net indebtedness of the private sector to the banking sector

where the negative sign preceding L indicates that loans from the banking sector represent a concomitant liability. If P_Y represents the average level of output prices then real net non-human wealth may be represented as follows:

$$\frac{W}{P_Y} = \frac{M}{P_Y} + \frac{T}{P_Y} + \left(\frac{P_b}{P_Y} \cdot B_p\right) + \left(\frac{P_k}{P_Y} \cdot K\right) - \frac{L}{P_Y} .$$

The Own Rates of Return

In order to investigate the wealth-adjustment process of this portfolio, the rates of return to these various assets must be specified. Each asset provides its holder with some form of return during the holding period; assuming the private sector exhibits no "money illusion," i.e., the demand for any real asset varies in response only to real variables and not to nominal values of these variables, the real rate of return of each asset is relevant to portfolio adjustment decisions.

Money does not provide its holder with an actual income stream; rather, it provides "income-in-kind," i.e., services rendered by the liquid nature of money. Therefore, the returns to money increase as the number of transactions and unforseen contingencies increase, i.e., Keynes' transaction and precautionary demand for money. The nominal return from a unit of real money during the holding period may be stated as follows:

i) $R_M = f_M(Y)$

where,

R_M = nominal rate of return from a unit of real money during the holding period

Y = real GNP

and R_M is positively related to Y. Since the analysis is concerned with the real returns, it must be noted that the real return to money during the holding period is decreased by inflation in the output sector. The real return from a unit of real money during the holding period may be stated as follows:

ii) $RR_M = f_M(Y) - \frac{\Delta P_Y}{P_Y}$ [1]

[1] In this study, the formulation of any real rate of return, e.g., RR_M, derived from a nominal rate of return to that asset is always

(footnote continued on next page)

where,

RR_M = real rate of return from a unit of real money during the holding period

γ = real GNP

$\frac{\Delta P_\gamma}{P_\gamma}$ = rate of change in output prices

and RR_M is positively related to γ and negatively related to $\frac{\Delta P_\gamma}{P_\gamma}$.

Returns to time deposits depend upon the current interest paid by financial institutions on this form of wealth. Also, individuals may hold time deposits in order to meet unforeseen contingencies, i.e., individuals may possess a precautionary demand for time deposits. Therefore, the nominal return from a unit of real time deposits may be stated as follows:

iii) $R_T = f_t(\gamma) + r_t$

where,

R_T = nominal rate of return from a unit of real time deposits during the holding period

r_t = nominal rate of interest paid upon time deposits

and R_T is positively related to γ. Since rising output prices tend

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(footnote continued from previous page) expressed as an approximation. Each real rate of return omits the cross-product of the nominal rate of return and the rate of change in output prices, e.g., $R_M \frac{\Delta P_\gamma}{P_\gamma}$. This component will always arise when calculating the purchasing power of an asset with a fixed nominal income. Omission of this component is consistent with other studies and recognizes that the term is ordinarily so small as to imply that its effect is inconsiderable.

to reduce the real return to time deposits during the holding period, the real return to time deposits during this period may be stated as follows:

$$\text{iv)} \quad RR_T = f_t(\gamma) + r_t - \frac{\Delta P_\gamma}{P_\gamma}^{1}$$

where,

RR_T = real rate of return from a unit of real time deposits during the holding period

and, again, RR_T is positively related to both γ and r_t; while RR_T is negatively related to $\frac{\Delta P_\gamma}{P_\gamma}$.

In order to analyze the returns to bonds and capital, both can be viewed as claims upon perpetual income streams. However, a bond has a <u>fixed</u> nominal income stream, while a unit of capital has a <u>variable</u> nominal income stream. This basic difference in the nature of the income streams generated by these assets forces the real rate of return from the two assets to possess different abilities to adjust to the changing output market condition, i.e., the real rate of return to capital will be said to "automatically" adjust to output market conditions. Using the convention of representing all bonds as consols, the fixed income stream of a bond is merely the periodic coupon return of the bond. The nominal return from a unit of real bonds during the holding period may be stated as follows:

$$\text{v)} \quad R_B = r_b = \frac{CR}{P_b}$$

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[1] RR_T is only an approximation, see footnote 1, page 17.

where,

R_B = nominal rate of return from a unit of real bonds during the holding period

r_b = nominal rate of interest paid upon a bond

CR = coupon return

P_b = average market price of a bond

and R_B is the discount rate which equates the income stream from the bond to its market price. Assuming the coupon return to be one dollar, the following equations may be easily derived:

(1) $P_b = \$1/r_b$

(2) $P_b/P_Y = \dfrac{\$1/P_Y}{r_b}$

(3) $r_b = \dfrac{\$1/P_Y}{P_b/P_Y}$

Equation 1 states the relationship between the market price of a bond and its discounted income stream. This relationship is merely restated in real terms in equation 2. Equation 3 defines the nominal rate of return to a bond as a function of its real income stream and the real value of the bond. Consider an increase in r_b, this can occur only if the real value of the income stream, $\$1/P_Y$, increases relative to the real value of the bond, P_b/P_Y. A bond has been defined as possessing a fixed nominal income stream; hence, the real income stream cannot vary independently of the real value of the bond. In other words, the real income stream of the bond can change only if output prices change, but output price changes also affect the real value of the bond. Since changes in the output sector cause proportionate changes in the real value of both the income stream and the bond, r_b will not reflect changes in P_Y. (Bonds are said not to "automatically" adjust to changes in the output sector.) Therefore, the nominal rate of return to bonds may rise <u>only</u> if the real value of bonds fall due to a fall in the nominal price of bonds. Thus, rising nominal rates of return to bonds imply that bondholders are worse off since the real value of bonds has fallen. Summarizing, the real return to a bondholder depends not only upon the nominal rate of return but also upon the rate of change in output prices and the rate of change in the nominal rate of return. The real rate of return from a unit of real bonds during the holding period may be stated as follows:

vi) $$RR_B = r_b - \frac{\Delta r_b}{r_b} - \frac{\Delta P_Y}{P_Y}$$ [1]

where,

RR_B = real rate of return from a unit of real bonds during the holding period

$\frac{\Delta r_b}{r_b}$ = rate of change of the nominal rate of return upon a bond

and RR_B is positively related to r_b and negatively related to both $\frac{\Delta r_b}{r_b}$ and $\frac{\Delta P_Y}{P_Y}$.

Capital can be viewed as an homogeneous asset providing its holder with a variable nominal income stream in perpetuity. The income stream

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[1] RR_B is only an approximation, see footnote 1, page 17.

generated by a unit of capital depends upon the marginal physical productivity of capital and the average level of output prices net of depreciation. The nominal return from a unit of real capital during the holding period may be stated as follows:

$$\text{vii)} \quad R_K = r_k = \frac{kP_Y^*}{P_k}$$

where,

R_K = nominal rate of return to a unit of real capital during the holding period

r_k = "nominal" rate of interest received by a unit of capital

k = marginal physical productivity of capital (net of depreciation)

P_Y^* = average level of output prices net of depreciation

and R_K is the discount rate which equates the income stream to the market price, P_k. A series of equations similar to those derived for bonds may also be derived for capital as follows:

$$(4) \quad P_k = \frac{k\,P_Y^*}{r_k}$$

$$(5) \quad P_k/P_Y = \frac{k\,P_Y^*/P_Y}{r_k}$$

assuming $P_Y^* = P_Y$,

$$(6) \quad r_k = \frac{k}{P_k/P_Y}\ .$$

Equation 4 states the relationship between the market price of a unit of capital and its discounted income stream. This relationship is merely restated in real terms in equation 5. Assuming $P_Y^* = P_Y$, equation 6 defines the "nominal" rate of return to capital as a function of the marginal physical productivity of capital (net of depreciation) and the real value of a unit of capital. Consider an increase in r_k, this can occur only if the real value of the income stream, k, rises relative to the real value of a unit of capital, P_k/P_Y. Unlike bonds, the income stream from capital is not rigid, i.e., it may change independently of output prices. Therefore increases in r_k represent ambiguous changes to the capital holder. If r_k rises due to increases in the real income stream, net productivity, then holders of capital are better off. However, if r_k rises due to decreases in the real value of a unit of capital, then they are worse off. Decreases in the real value of a unit of capital, P_k/P_Y, may occur due to either decreases in the market price of capital or increases in output prices. In the latter case, the real income stream will be unaffected unless the marginal physical productivity also changes. The "nominal" rate of return will "automatically" adjust to output price changes; hence, this rate of return is better named the real rate of return to capital. Summarizing, the real return to capital depends solely upon the real rate of return to capital and changes in this rate induced by either changes in productivity or the market price of capital. The real return from a unit of real capital during the holding period may be stated as follows:

viii) $$RR_K = r_k + \frac{\Delta k}{k} - \frac{\Delta(P_k/P_Y)}{P_k/P_Y}$$

where,

RR_K = real rate of return to a unit of real capital during the holding period

$\frac{\Delta k}{k}$ = rate of change in the marginal physical productivity of capital (net of depreciation)

$\frac{\Delta(P_k/P_Y)}{P_k/P_Y}$ = rate of change in the real value of a unit of capital

and RR_K is positively related to r_k and $\frac{\Delta k}{k}$, while negatively related to $\frac{\Delta(P_k/P_Y)}{P_k/P_Y}$. Assuming that productivity does not change during the period of analysis then the real return from a unit of real capital during the holding period becomes:

ix) $$RR_K = r_k - \frac{\Delta(P_k/P_Y)}{P_k/P_Y}.$$

The discussion of the returns to the assets contained in the portfolio is not complete without a discussion of the return to loans from the banking sector. As previously stated, the net indebtedness of the private sector to the banking sector represents a concomitant liability which must be taken into consideration in the calculation of net non-human wealth of the private sector. Obviously, the return to this liability is negative when viewed in context with the other assets. The nominal return from a unit of real loans from the banking sector is fixed by contractual agreement over the holding period and may be stated as follows:

x) $R_L = -r_L$

where,

R_L = nominal rate of return from a unit of real loans from the banking sector

r_L = rate of interest charged by banking system for borrowing

and R_L is negatively related to r_L. Since net debtors benefit from inflation, the real return from a unit of real loans from the banking sector during the holding period may be stated as follows:

xi) $RR_L = \frac{\Delta P_Y}{P_Y} - r_L$.[1]

Portfolio Model

Recalling the above considerations, a portfolio model of the net non-human wealth of the private sector may be stated in terms of the demand equations for these various assets. Utilizing Tobin's warning concerning cross-effects, i.e., $\sum_{i=1}^{5} \frac{\partial f_i}{\partial RR_k} = 0$, where f_i represents the demand for the i^{th} asset, then the entire list of relevant interest rates should appear in each demand function. Even if the cross-effects should appear insignificant empirically, an interest rate should not be omitted since the sum of the cross-effects are equal in absolute terms

[1] RR_L is an approximation, see footnote 1, page 17.

to the magnitude of the own-effect. This is true for all variables which enter into the demand functions with the exception of net non-human wealth. The sum of the asset changes brought about by a change in net non-human wealth must equal one, i.e., $\sum_{i=1}^{5} \frac{\partial f_i}{\partial W} = 1$. Also, the asset demand functions stated in real terms are homogeneous of degree zero in output prices and nominal wealth.

The Portfolio with Income

The real demand for the assets defined above may be stated as follows:

1a. $$\frac{M^d}{P_Y} = f_1(Y, \frac{\Delta P_Y}{P_Y}, r_t, r_b, \frac{\Delta r_b}{r_b}, r_k, \frac{\Delta (P_k/P_Y)}{P_k/P_Y}, r_L, \frac{W}{P_Y})$$

2a. $$\frac{T^d}{P_Y} = f_2(Y, \frac{\Delta P_Y}{P_Y}, r_t, r_b, \frac{\Delta r_b}{r_b}, r_k, \frac{\Delta (P_k/P_Y)}{P_k/P_Y}, r_L, \frac{W}{P_Y})$$

3a. $$\frac{(B_p P_b)^d}{P_Y} = f_3(Y, \frac{\Delta P_Y}{P_Y}, r_t, r_b, \frac{\Delta r_b}{r_b}, r_k, \frac{\Delta (P_k/P_Y)}{P_k/P_Y}, r_L, \frac{W}{P_Y})$$

4a. $$\frac{(KP_k)^d}{P_Y} = f_4(Y, \frac{\Delta P_Y}{P_Y}, r_t, r_b, \frac{\Delta r_b}{r_b}, r_k, \frac{\Delta (P_k/P_Y)}{P_k/P_Y}, r_L, \frac{W}{P_Y})$$

5a. $$\frac{L^d}{P_Y} = f_5(Y, \frac{\Delta P_Y}{P_Y}, r_t, r_b, \frac{\Delta r_b}{r_b}, r_k, \frac{\Delta (P_k/P_Y)}{P_k/P_Y}, r_L, \frac{W}{P_Y})$$

and for each f_i, i=1, ..., 5, the quantity demanded is positively related to those components of its own real rate of return which tend to increase this rate and negatively related to those components which tend to reduce the own real rate. The opposite results are expected in regard to other real rates. Assuming that all these asset demands are linear in logarithms,[1] the above equations may be rewritten as follows:

$$\text{1b.}\quad \ln \frac{M^d}{P_\gamma} = \ln \alpha_0 + \alpha_1 \ln \gamma + \alpha_2 \ln \frac{P_{\gamma(t)}}{P_{\gamma(t-1)}} + \alpha_3 \ln r_t + \alpha_4 \ln r_b + \alpha_5 \ln \frac{r_{b(t)}}{r_{b(t-1)}} + \alpha_6 \ln r_k + \alpha_7 \ln \frac{(P_k/P_\gamma)_{(t)}}{(P_k/P_\gamma)_{(t-1)}} + \alpha_8 \ln r_L + \alpha_9 \ln \frac{W}{P_\gamma}$$

$$\text{2b.}\quad \ln \frac{T^d}{P_\gamma} = \ln \beta_0 + \beta_1 \ln \gamma + \quad . \quad . \quad . \quad + \beta_9 \ln \frac{W}{P_\gamma}$$

$$\text{3b.}\quad \ln \frac{(B_p P_b)^d}{P_\gamma} = \ln \mu_0 + \mu_1 \ln \gamma + \quad . \quad . \quad . \quad + \mu_9 \ln \frac{W}{P_\gamma}$$

$$\text{4b.}\quad \ln \frac{(K P_k)^d}{P_\gamma} = \ln \eta_0 + \eta_1 \ln \gamma + \quad . \quad . \quad . \quad + \eta_9 \ln \frac{W}{P_\gamma}$$

$$\text{5b.}\quad \ln \frac{L}{P_\gamma} = \ln \lambda_0 + \lambda_1 \ln \gamma + \quad . \quad . \quad . \quad + \lambda_9 \ln \frac{W}{P_\gamma}$$

---

[1]In order to avoid quite restrictive assumptions, i.e., average relationships change but marginal relationships do not, about the nature of the demands for assets, the log-linear functional form was chosen for all equations. This functional form allows immediate comparisons between the results of this study and the majority of empirical evidence collected in this area. In other words, the estimates of the coefficients of the independent variables represent elasticities which are assumed to be constant throughout the range of a given demand function. Most empirical evidence in this area consists of reporting and comparing such elasticities.

Since the log-linear form has been chosen for all demand functions, substitutions must be made for all rates of change. That is, $\frac{\Delta P_\gamma}{P_\gamma}$, $\frac{\Delta r_b}{r_b}$, and $\frac{\Delta(P_k/P_\gamma)}{P_k/P_\gamma}$ may be replaced by $\frac{P_\gamma(t)}{P_\gamma(t-1)}$, $\frac{r_b(t)}{r_b(t-1)}$, and $\frac{(P_k/P_\gamma)(t)}{(P_k/P_\gamma)(t-1)}$. This is necessary due to the possibility that no change and/or a negative change might occur over a given period, hence, it would be impossible to take the logarithms of such changes. At any point in time, portfolio equilibrium requires that the demand for each asset be equal to the observed supply of each asset, which may be stated as follows:

6. $M^d = M^s$

7. $T^d = T^s$

8. $B_p^d = B_p^s$

9. $K^d = K^s$

10. $L^d = L^s$

where the "s" indicates the observed supply. The demand for the various assets can be estimated by substituting the appropriate demand function into the equilibrium condition. (Although this method was chosen in order to only necessitate the use of ordinary-least-squares regression, it is recognized that the system is simultaneous.)

However, it should be noted that only four of these equilibrium conditions need be satisfied in order that all markets be in equilibrium.

In other words, it is only necessary to estimate the demands for any four assets, if any four markets are in equilibrium the fifth must necessarily be in equilibrium due to their sum being defined as total net non-human wealth of the private sector. Also, it should be noted that several "changes" in components of the real rates of return appear in each demand function. Since the above analysis represents a stock equilibrium, these "changes" must be given an interpretation consistent with this form of analysis. The convention adopted by this study is to interpret "changes" as expected or anticipated changes that might occur during the holding period. Hence, the appropriate real returns in the analysis are those calculated upon the basis of these expectations and perceived to be "correct."

The Portfolio without Income

Recalling the remarks concerning the real rates of return to the various assets, a slight revision of the above model may provide additional insight. An important methodological problem has been introduced into the above analysis. Namely, considering the relationships among wealth, interest, and income, should all three be simultaneously included as independent variables within the same function? In other words, "the" interest rate may be viewed as the discount rate which equates the flow of income to the stock of wealth. If wealth and interest are key explanatory variables within a function, should income also appear as an independent variable? Because of these considerations and the fact that income enters the model only as a proxy for a real rate of return, it seems advisable to reformulate the above demand equations omitting income as an explanatory variable.

$$1c.\quad \ln \frac{M^d}{P_\gamma} = \ln \alpha_0 + \alpha_1 \ln \frac{P_{\gamma(t)}}{P_{\gamma(t-1)}} + \alpha_2 \ln r_t + \alpha_3 \ln r_b + \alpha_4 \ln \frac{r_{b(t)}}{r_{b(t-1)}} + \alpha_5 \ln r_k + \alpha_6 \ln \frac{(P_k/P_\gamma)_{(t)}}{(P_k/P_\gamma)_{(t-1)}} + \alpha_7 \ln r_L + \alpha_8 \ln \frac{W}{P_\gamma}$$

$$2c.\quad \ln \frac{T^d}{P_\gamma} = \ln \beta_0 + \beta_1 \ln \frac{P_{\gamma(t)}}{P_{\gamma(t-1)}} + \ldots + \beta_8 \ln \frac{W}{P_\gamma}$$

$$3c.\quad \ln \frac{(B_p P_b)^d}{P_\gamma} = \ln \mu_0 + \mu_1 \ln \frac{P_{\gamma(t)}}{P_{\gamma(t-1)}} + \ldots + \mu_8 \ln \frac{W}{P_\gamma}$$

$$4c.\quad \ln \frac{(K\ P_k)^d}{P_\gamma} = \ln \eta_0 + \eta_1 \ln \frac{P_{\gamma(t)}}{P_{\gamma(t-1)}} + \ldots + \eta_8 \ln \frac{W}{P_\gamma}$$

$$5c.\quad \ln \frac{L}{P_\gamma} = \ln \lambda_0 + \lambda_1 \ln \frac{P_{\gamma(t)}}{P_{\gamma(t-1)}} + \ldots + \lambda_8 \ln \frac{W}{P_\gamma}$$

Empirical results from estimating both portfolio formulations will be presented in the next chapter.

Expectations and Distributed Lags

Evidence regarding the formulation of expectations regarding the performance of economic variables appears quite sparse. Anticipated "changes" in economic variables are probably influenced by: (1) the immediate past performance of this variable; (2) the historical behavior of this variable; (3) the error-learning process in which individuals engage; and (4) the institutional changes which occur during the expectation formulating period. An attempt to capture these various influences leads most authors to formulating expected variables as weighted averages of past values of the variable, e.g.,

$$X_t^* = \sum_{i=0}^{n} \sigma_i X_{t-i}$$

where,

X^* = expected value of X $\qquad\qquad$ $\sigma_0 > \sigma_1 > \ldots > \sigma_n$.

This study adopts a polynomial distributed lag technique as a means of estimating expected values. The distributed lag technique used was that proposed by Shirley Almon (1965), namely, the method of Lagrange for polynomial interpolation combined with ordinary least squares estimation technique. Relevant statistical and technical aspects of the Almon technique are discussed in the appendix.

In order to implement this technique, a criterion for determining the desired lag length to be used in conjunction with each lagged independent variable must be accepted. Leonall Anderson (1969) has proposed that the proper criterion should be the standard error of the overall estimate of the reduced form. Obviously, this study does not estimate a reduced form of a model. Therefore, this study adopted the following criterion: assuming the lag length for a particular expected variable should be the same in all demand functions, the desired lag length can be found by minimizing the standard error of the overall estimate of the lagged-variable's <u>own</u> demand function. An attempt was also made to allow the lag structure to approach zero in latter quarters so that no additional explanation of the variance of the dependent variable was lost.

Formulation of the equations to be estimated to include the perceived real rates of return may be accomplished by incorporating the lagged independent variables in the following form:

$$11.\quad \ln \frac{M^d}{P_\gamma} = \ln \alpha_0 + \ln \gamma^* + \ln \frac{P_{\gamma(t)}}{P_{\gamma(t-1)}}^* + \ln r_t^*$$

$$+ \ln r_b^* + \ln \frac{r_{b(t)}}{r_{b(t-1)}}^* + \ln r_k^*$$

$$+ \ln \frac{(P_k/P_\gamma)_{(t)}}{(P_k/P_\gamma)_{(t-1)}}^* + \ln r_L^* + \ln \frac{W}{P_\gamma}$$

where * means expected, and $\ln X_{j,t}^* = \sum_{i=0}^{m_j} \alpha_j w_j(i) \ln X_{j,t-i}$, where

$j=1, \ldots, 8$. The weights are summed over the lag length of the independent variable and the product of the α's and w(i) represent the pattern of expectation formulation. Due to the serious serial correlation of regression residuals encountered in initial estimations, an iterative generalized-least-squares technique was adopted.

CHAPTER III. EMPIRICAL RESULTS

Before examining the empirical results obtained from the estimates of the portfolio models, it is necessary to state the criterion upon which the results may be judged. Obviously, the number of correct a priori signs or, more specifically, the number of significant coefficients possessing correct a priori signs may provide a valid criterion. However, judgments based solely upon such criteria might cause empirical tests of portfolio results to be falsely accused of yielding dismal results. The degree of aggregation and the number of non-observable variables inherent in such tests may contribute greatly to such results. Therefore, the above criteria may be amended to place emphasis upon those demand equations within the portfolio whose estimation utilized the most robust data. The above considerations lead to an emphasis upon the demand for real money and the demand for real time deposits. (It should be noted that this emphasis is consistent with the role that these assets occupy in current issues in monetary economics.) Recalling the comments in the preceding chapter concerning the number of equations within the portfolio which must be estimated, the demand for capital was not estimated. This equation was omitted due to its data exhibiting the highest degree of aggregation and the lack of well defined capital markets.

The Portfolio with Income

All four demand equations were fit utilizing a third degree polynomial in the estimation of the distributed lag function. The Almon technique for estimating distributed-lag weight structures does not require the usual a priori assumption of a particular shape for the weight structure. It requires only that the shape of the weight structure can be approximated well by a polynomial of some specified degree. Hence, the third degree polynomial was chosen after attempts to use various other polynomials and lag lengths were rejected due to loss of degrees of freedom or the additional computation costs. Since each expected real return appears in all four equations, each expected variable was restricted to the same lag length in all equations, i.e., if r_k was restricted to a four period lag in one equation, it was restricted to a four period lag in all four equations. In other words, individuals within the private sector formulate expectations, e.g., lag length, concerning each real rate of return and they do not alter this expected value depending upon which asset they are concentrating. This does not restrict individuals within the sector to formulate all expectations of various rates in the same way. Various lag lengths as well as discrete lags allow for the formulation of expectations of various rates to occur independently.

Arbitrary periods selected for the interpolation of the polynomial lag structure which extended four quarters in the past were 0.0, 2.0, 3.0, and 4.0. For the eleven quarter lag, they were 0.0, 2.0, 4.0, and 11.0. Although these lags were selected arbitrarily, in both cases the last quarter in the lag was one of the selected periods. This restriction provided program efficiency in the final computations.

Early attempts to estimate the importance of the rate of change in the real value of a unit of capital, $\frac{(P_k/P_Y)_{(t)}}{(P_k/P_Y)_{(t-1)}}$, in the determination of the demand for these assets resulted in no additional information as illustrated in Appendix C. In all equations, this variable appears insignificant and the correct a priori sign occurs only in the net indebtedness equation. Two possible explanations for these results are: (1) the stockmarket provides a poor indicator for changes in the real value of capital, much of which is not represented by stockmarket transactions, e.g., housing, and (2) portfolio decisions are not influenced by changes in the real value of capital, especially since there are no well-defined and organized capital markets. In any case, it was deemed proper to omit the rate of change of the real value of capital as an explanatory variable within the portfolio.

Estimation of two expected variables using the above technique resulted in additional information with the introduction of a discrete lag. Both the nominal rate of return to capital and the rate of interest charged by the banking system on loans perform better with the introduction of a two and three period discrete lag respectively. In both cases, the introduction of a discrete lag at the beginning of the distributed lag function merely reflects the way in which individuals within the private sector formulate expectations. A similar consideration of expectations formulation, led to the introduction of the nominal rate of return to bonds and real income as single period variables. It appears that individuals within the private sector form return to bonds expectations on the basis of the immediate past, i.e., the expected nominal rate of return to bonds equals r_b (t-1). Income expectations appear to encompass only the current value of real income. It should be noted that real net wealth appears as a single current period variable because it represents the constraint upon portfolio allocation and that it is not an expected value.

As previously mentioned, it was necessary to use only two lag lengths in estimating the distributed lag function. The following variables were estimated utilizing the four period lag: the nominal rate of return to capital, r_k; the rate of change of the nominal rate of return to a bond, $\frac{r_b (t)}{r_b (t-1)}$; the rate of interest charged by the banking system on loans, r_L; and the rate of change in output prices, $\frac{P_Y (t)}{P_Y (t-1)}$. Only the nominal rate of interest paid on time deposits, r_t, required the usage of the eleven period lag.

It should be noted that, with the exception of the nominal rate of return to bonds, all market-determined prices and yields possess a short expectations formulation base. This base period, which coincides with a one year period, appears to be too short to allow expectations to be formulated concerning institutional changes. The nominal rate of interest paid on time deposits was proxied by the ceiling rate placed upon these accounts by the Federal Reserve System. Apparently, individuals within the private sector observe more of the past when the observed variable is institutionally-determined rather than market-determined. This result may be more pronounced due to the historical immobility of the ceiling rate.

Due to the serial correlation encountered, two iterations of the generalized least-squares routine was employed. The results obtained from estimating the demand equations within the portfolio with income

Table 3.1. Generalized-least-squares regression results for the demand for real money,[a] 1949-I through 1969-IV

Dependent variable: $\ln M^s/P_\gamma$

Independent variables:	$\ln \gamma_t$	$\ln \frac{P_\gamma(t)^*}{P_\gamma(t-1)}$	$\ln r_t^*$	$\ln r_{b(t-1)}$	$\ln \frac{r_b(t)^*}{r_b(t-1)}$	$\ln r^*_{k(t-2)}$	$\ln r^*_{L(t-3)}$	$\ln \frac{W}{P_\gamma}_t$
Coefficients:	.204 (3.702)	-1.210 (-1.925)	-.092 (-1.894)	-.056 (-4.844)	.200 (3.307)	.126 (7.450)	-.075 (-2.589)	.442 (3.702)
Weights:								
w(0)		.176 (1.662)	.028 (.202)		.035 (.550)	.272 (6.356)	.155 (.712)	
w(1)		.141 (1.162)	.312 (2.124)		.483 (7.286)	.270 (8.856)	.542 (2.637)	
w(2)		.312 (5.891)	.439 (2.117)		.382 (11.852)	.264 (12.361)	.323 (2.962)	
w(3)		.372 (2.691)	.442 (2.137)		.099 (1.572)	.194 (5.348)	-.019 (-.093)	
w(4)		.000	.356 (2.285)		.000	.000	.000	
w(5)			.212 (2.770)					

w(6)	.043 (.769)
w(7)	-.116 (-.857)
w(8)	-.233 (-1.190)
w(9)	-.275 (-1.321)
w(10)	-.208 (-1.389)
w(11)	.000

[a]Figures in parentheses are t-statistics, the intercept is .094, $\bar{R}^2$ = .765, Durbin-Watson = 1.65, ρ = .700, and ρ' = .455.

Table 3.2. Generalized-least-squares regression results for the demand for real time deposits,[a] 1949-I through 1969-IV

Dependent variable: $\ln T^s/P_Y$

Independent variables:	$\ln Y_t$	$\ln \frac{P_Y(t)}{P_Y(t-1)}^*$	$\ln r_t^*$	$\ln r_{b(t-1)}$	$\ln \frac{r_b(t)}{r_b(t-1)}^*$	$\ln r_{k(t-2)}^*$	$\ln r_{L(t-3)}^*$	$\ln \frac{W}{P_Y}_t$
Coefficients:	.513 (2.808)	-2.367 (-1.294)	.931 (6.876)	-.158 (-4.658)	-.206 (-1.160)	.082 (1.743)	-.038 (-.467)	1.486 (4.311)
Weights:								
w(0)		.130 (.764)	.143 (3.573)		.484 (1.245)	.452 (1.946)	.802 (.501)	
w(1)		-.025 (-.088	.113 (5.248)		-.118 (-.290)	.108 (.641)	1.786 (.519)	
w(2)		.335 (3.923)	.096 (4.581)		.158 (.813)	.174 (1.500)	-.001 (-.001)	
w(3)		.560 (1.737)	.088 (3.930)		.476 (1.875)	.266 (1.593)	-1.587 (-.400)	
w(4)		.000	.087 (4.350)		.000	.000	.000	
w(5)			.089 (5.933)					

w(6)	.092
	(7.504)
w(7)	.092
	(5.964)
w(8)	.086
	(4.306)
w(9)	.071
	(3.379)
w(10)	.043
	(2.845)
w(11)	.000

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[a]Figures in parentheses are t-statistics, the intercept is -1.913, $\bar{R}^2$ = .977, Durbin-Watson = 1.54, ρ = .683, and ρ' = .424.

Table 3.3. Generalized-least-squares regression results for the demand for real bond holdings,[a] 1949-I through 1969-IV

Dependent variable: $\ln B_p^s/P_Y$

Independent variables:	$\ln Y_t$	$\ln \frac{P_{Y(t)}}{P_{Y(t-1)}}^*$	$\ln r_t^*$	$\ln r_{b(t-1)}$	$\ln \frac{r_{b(t)}}{r_{b(t-1)}}^*$	$\ln r_{k(t-2)}^*$	$\ln r_{L(t-3)}^*$	$\ln \frac{W}{P_Y}_t$
Coefficients:	-.284 (-1.200)	-6.938 (-2.947)	.035 (.259)	-.023 (-.494)	-.148 (-.607)	-.211 (-4.184)	-.226 (-2.603)	.466 (1.104)
Weights:								
w(0)		.145 (1.567)	-1.722 (-.231)		.510 (.616)	.043 (.343)	.422 (1.373)	
w(1)		.365 (4.571)	-.450 (-.208)		-.485 (-.354)	.127 (1.383)	.651 (2.067)	
w(2)		.328 (7.091)	.359 (.272)		.145 (.351)	.379 (6.079)	.189 (1.232)	
w(3)		.162 (1.966)	.784 (.264)		.830 (.815)	.451 (4.118)	-.262 (-.834)	
w(4)		.000	.907 (.267)		.000	.000	.000	
w(5)			.806 (.278)					

w(6)	.563
	(.305)
w(7)	.256
	(.392)
w(8)	-.034
	(-.042)
w(9)	-.226
	(-.157)
w(10)	-.242
	(-.185)
w(11)	.000

[a]Figures in parentheses are t-statistics, the intercept is 1.584, $\overline{R}^2$ = .567, Durbin-Watson = 1.67, ρ = .478, and ρ' = .270.

Table 3.4. Generalized-least-squares regression results for the demand for real loans,[a] 1949-I through 1969-IV

Dependent variable: $\ln L^S/P_Y$

Independent variables:	$\ln Y_t$	$\ln \frac{P_Y(t)}{P_Y(t-1)}^*$	$\ln r_t^*$	$\ln r_{b(t-1)}$	$\ln \frac{r_b(t)}{r_b(t-1)}^*$	$\ln r^*_{k(t-2)}$	$\ln r^*_{L(t-3)}$	$\ln \frac{W}{P_Y}_t$
Coefficients:	.611 (4.339)	1.915 (1.367)	.440 (4.784)	-.015 (.545)	-.202 (-1.444)	.062 (1.893)	-.047 (-.805)	1.343 (5.200)
Weights:								
w(0)		.304 (1.691)	.100 (1.539)		.236 (1.256)	.440 (1.839)	-.145 (-.142)	
w(1)		.670 (2.179)	.087 (2.808)		.230 (1.778)	.106 (.609)	1.232 (.869)	
w(2)		.248 (2.758)	.083 (2.782)		.282 (3.006)	.180 (1.504)	.472 (.923)	
w(3)		-.222 (-.639)	.087 (2.506)		.252 (2.149)	.274 (1.503)	-.559 (-.474)	
w(4)		.000	.094 (2.904)		.000	.000	.000	
w(5)			.102 (4.093)					

w(6)	.108
	(5.755)
w(7)	.108
	(5.034)
w(8)	.101
	(3.567)
w(9)	.082
	(2.713)
w(10)	.049
	(2.224)
w(11)	.000

[a]Figures in parentheses are t-statistics, the intercept is -2.301, $\bar{R}^2$ = .990, Durbin-Watson = 1.65, ρ = .606, and ρ' = .357.

are presented in Tables 3.1 to 3.4. All but six of the thirty-two estimated coefficients possess the correct a priori sign. Eighteen coefficients possessing the correct a priori sign are significant at the .05 level.

The most impressive results were obtained from the estimation of the real demand for money. All the independent variables possess the correct a priori sign and are statistically significant with the single exception of the rate of return to capital. This extremely puzzling result is compounded by the fact that the rate of return to capital appears to be highly significant and that it also possesses an incorrect a priori sign in the demand for real time deposits. One possible explanation is that the proxy for the return to capital, i.e., earnings-price ratio, may not adequately reflect the vector of rates of return to capital. Excluding the possibility that this result is merely due to the specific measure chosen, no plausible explanation suggests itself.

A striking feature of the real demand for money is the relative insensitivity to interest rates. While all rates of return appear to be quite significant, the coefficients suggest that the real demand for money is relatively unresponsive to changes in any single rate of return. However, in order to compare these results with previous empirical investigations, it should be noted that the introduction of a single rate of return into the demand for money is often justified by contending that one rate of return represents the entire vector of relevant rates. Hence, assuming that the relevant rates of return tend to move together, any comparison of the interest-elasticity requires that individual elasticities be summed, i.e., a one percent simultaneous increase in the various returns composing the vector should be contrasted to a one percent change in the single rate of return which proxies such a vector. This suggests that the elasticity coefficients for rates of return to time deposits, bonds, and loans should be summed in order to compare with previous results. Doing so, an interest-elasticity coefficient of -.223 results which easily falls into the range of previous results. It should be noted that the rate of return to capital was excluded from this summation. This was done not only in light of the incorrect a priori sign but also, because it was believed that this return does not move together with those rates of return which represent assets possessing a fixed nominal income stream.

Possibly the most interesting result of estimating the real demand for money is the sensitivity of this demand to fluctuations in the rate of return to bonds. Again, the relative responsiveness of the real demand for money is small. However, the importance of the statistical significance of the rate of change in the bond rate is twofold: (1) it suggests a speculative motive for holding real money balances and (2) it suggests an extrapolative expectations formulation. Considering that the rate of change in the bond rate was statistically significant only in the demand for money balances, no strong case can be made for a speculative motive within the portfolio. Nevertheless, this may suggest that models which specify the expected returns to assets as important explanatory variables are more likely to encounter such a motive. The negative sign of this coefficient suggests that individuals expect the direction of recent past changes in rates of return will continue in the future. That is, if the rate of return on bonds rises, it is expected that it will continue to rise in the future. Therefore, changes in the rate of return to bonds are expected to produce capital losses when this rate of return rises and vice versa. These expectations provide the motivation for speculation.

The real demand for money is more sensitive to changes in output prices than to any other explanatory variable. A one percent change in the change in output prices is accompanied by a 1.2 percent decrease in the demand for real money balances.

Both real income and real wealth appear as statistically significant independent variables within the real demand for money function. While the real demand for money appears to be twice as responsive to a one percent change in real wealth than to a one percent change in real income, the demand for real money balances appears to be relatively unresponsive to either. Certainly, the elasticity coefficient of real wealth, .442, does not suggest that the real demand for money is linearly homogeneous in real wealth as some authors have hypothesized.

The results of estimating the real demand for time deposits was less impressive than those obtained by estimating the real demand for money although the adjusted coefficient of determination, $\bar{R}^2$, suggests that the model explains more of the variation in the real demand for time deposits, i.e., $\bar{R}^2$ = .977 contrasted to $\bar{R}^2$ = .765. While six of the eight independent variables possess the correct a priori sign only four of these are statistically significant at the .05 level. Three rates of return appear to be statistically significant. As previously mentioned the rate of return to capital possesses an incorrect a priori sign, but both the rate of return to bonds and the rate of return to time deposits appear to be highly significant while possessing the correct a priori sign. As might be anticipated, the real demand for time deposits is relatively highly responsive to the rate of return on time deposits, i.e., elasticity coefficient of .931. Also, the real demand for time deposits is relatively unresponsive to the rate of return upon bonds. The lack of statistical significance of the rate of change in the bond rate may suggest that time deposits are not as liquid as money balances and reinforce those arguments concerning the acceptance of a narrow definition of money as proper in analysis of monetary phenomenas.

Although the rate of interest charged by the banking system upon loans appears to have the correct a priori sign, it may not be too surprising to note that it is not statistically significant. It is quite doubtful that the same individuals who acquire loans from the banking sector hold a large share of the time deposits. In other words, it is unlikely that an individual would borrow funds in order to place them into some form of savings.

Indeed, it is more interesting to note the statistical insignificance of the change in output prices. One possible explanation for the statistical insignificance of the change in output prices rests with the choice of the lag length of this particular independent variable. The criteria for choosing the lag length relied upon viewing each return within its own demand equation. However, the change in output prices is a component of several real rates of return. Hence, the appropriate lag length was chosen by viewing the change in output prices in all four equations with primary emphasis being placed upon the demand for money equation. Therefore, the poor performance of this variable may be contributed to the particular lag structure chosen rather than to its explanatory ability.

Both real income and real wealth appear statistically significant and possess the correct a priori sign. While both real income and wealth appear to be quite significant, it is interesting to note the relative responsiveness of the real demand for time deposits. The real demand for time deposits appear to be more responsive to changes in income than does the real demand for money. Furthermore, the real demand for time deposits appears to be quite responsive to changes in real wealth, i.e., elasticity coefficient of 1.5.

The remaining demand equations, the real demand for bonds and the real demand for loans, yield less impressive results in terms of the number of significant variables when compared to the two previous demand equations. Aside from the previously mentioned incorrect a priori sign which accompanies the rate of return to capital in those equations just discussed, the most disappointing result is the incorrect a priori sign attached to the rate of return to bonds in its own equation. Only two signs appear a priori to be incorrect in the real demand for bonds, including the rate of return to time deposits. Both, however, appear to be statistically insignificant. It can be argued that the rate of return to bonds appears to possess the incorrect sign in part due to the number of insignificant variables which appear in this equation. Along with the aforementioned, the change in the bond rate, real wealth, and real income appear statistically insignificant although possessing the correct a priori sign.

Indeed, the choice of the lag structure for the rate of return for bonds as well as the change in this rate would seem dubious considering their lack of significance within their own equation. Again, this judgment was not made solely upon this single formulation. The criteria for choosing the lag structure was modified to take into consideration several alternative formulations and the overall performance of each independent variable.

One cheerful note was the statistical significance of the rate of return to capital which possesses the correct a priori sign in the real demand for bonds. It is also interesting to note the extremely high elasticity coefficient with respect to changes in output prices, i.e., elasticity coefficient of 7.0. Again, this result may simply reflect the poor performance of this entire equation.

Seven of the independent variables in the real demand for loans possess the correct a priori sign, but only four of these appear statistically significant. Again, the difficulty encountered in choosing the lag structure for the own rate may have attributed to the insignificance of the rate of interest charged to individuals for borrowing from the banking sector. The insignificance of two components of the real return to bonds may have been anticipated as previously mentioned and tends to reinforce this explanation. Both the rate of return to capital and the rate of return to time deposits appear statistically significant and possess the correct a priori sign. Again, the insignificance of the change in output prices may be due to the lag structure chosen. Both real income and real wealth possess the correct a priori sign and are statistically significant. The real demand for loans appears to be much more responsive to real wealth than to real income.

The Portfolio without Income

Using the same procedure and lag structures as described above, the four demand equations were estimated omitting real income as an independent variable. The results obtained from estimating the demand equations within a portfolio without income are presented in Tables 3.5 to 3.8. All but five of the twenty-eight estimated coefficients possess the correct a priori sign. Twelve coefficients possessing the correct a priori sign are significant at the .05 level. Compared with the previous portfolio specification there was apparently <u>no loss</u> of any additional information by omitting real income from these equations. Only one sign changed (the rate of return to time deposits in the real demand for bonds equation) and this variable remained statistically insignificant. Four significant variables in the portfolio with income became statistically insignificant with the omission of real income.

Both the rate of return to time deposits and the change in output prices became statistically insignificant in the real demand for money equation. While all other significant coefficients remained in the same order of magnitude, the elasticity coefficient for real wealth increased from .442 to .684 and became highly significant. The rate of return to capital became insignificant in both the real demand for time deposits and for loans. Again, the same pattern of behavior was noted

Table 3.5. Generalized-least-squares regression results for the demand for real money,[a] 1949-I through 1969-IV

Dependent variable: $\ln M^s/P_Y$

Independent variables:	$\ln \frac{P_Y(t)}{P_Y(t-1)}^*$	$\ln r_t^*$	$\ln r_{b(t-1)}$	$\ln \frac{r_b(t)}{r_b(t-1)}^*$	$\ln r_{k(t-2)}^*$	$\ln r_{L(t-3)}^*$	$\ln \frac{W}{P_Y}_t$
Coefficients:	-.968 (-1,401)	-.031 (-.672)	-.059 (-4.628)	.267 (4.123)	.133 (7.544)	-.089 (-2.936)	.684 (6.841)
Weights:							
w(0)	.156 (1.018)	-.088 (-.168)		.084 (1.620)	.279 (6.111)	.163 (.790)	
w(1)	.017 (.074)	.836 (.728)		.444 (9.215)	.278 (8.489)	.454 (2.578)	
w(2)	.322 (4.205)	1.239 (.710)		.358 (13.786)	.260 (11.382)	.319 (3.088)	
w(3)	.505 (1.939)	1.232 (.712)		.114 (2.250)	.183 (4.689)	.064 (.347)	
w(4)	.000	.929 (.732)		.000	.000	.000	
w(5)		.443 (.827)					

w(6)	-.114 (-.317)
w(7)	-.628 (-.563)
w(8)	-.986 (-.601)
w(9)	-1.076 (-.615)
w(10)	-.785 (-.623)
w(11)	.000

[a]Figures in parentheses are t-statistics, the intercept is .009, $\bar{R}^2$ = .761, Durbin-Watson = 1.55, ρ = .659, and ρ' = .441.

Table 3.6. Generalized-least-squares regression results for the demand for real time deposits,[a] 1949-I through 1969-IV

Dependent variable: $\ln T^s/P_Y$

Independent variables:	$\ln \frac{P_Y(t)}{P_Y(t-1)}^*$	$\ln r_t^*$	$\ln r_{b(t-1)}$	$\ln \frac{r_b(t)}{r_{b(t-1)}}^*$	$\ln r_{k(t-2)}^*$	$\ln r_{L(t-3)}^*$	$\ln \frac{W}{P_Y}_t$
Coefficients:	-1.872 (-.968)	1.101 (8.099)	-.159 (-4.438)	-.063 (-.347)	.081 (1.586)	-.074 (-.837)	2.029 (7.061)
Weights:							
w(0)	.105 (.450)	.126 (3.555)		1.020 (.365)	.478 (1.842)	.478 (.688)	
w(1)	-.204 (-.369)	.100 (5.239)		-1.057 (-.263)	.136 (.795)	.839 (.961)	
w(2)	.348 (2.977)	.088 (4.632)		-.110 (-.079)	.161 (1.241)	.161 (.463)	
w(3)	.751 (1.232)	.085 (4.170)		1.146 (.429)	.225 (1.291)	-.479 (-.481)	
w(4)	.000	.088 (4.870)		.000	.000	.000	
w(5)		.094 (6.975)					

w(6)	.094
	(9.098)
w(7)	.100
	(7.218)
w(8)	.095
	(5.219)
w(9)	.078
	(4.107)
w(10)	.048
	(3.465)
w(11)	.000

[a]Figures in parentheses are t-statistics, the intercept is -1.968, $\bar{R}^2$ = .972, Durbin-Watson = 1.47, ρ =.655, and ρ' = .496.

Table 3.7. Generalized-least-squares regression results for the demand for real bond holdings,[a] 1949-I through 1969-IV

Dependent variable: $\ln B_p^s/P_Y$

Independent variables:	$\ln \frac{P_{Y(t)}}{P_{Y(t-1)}}^*$	$\ln r_t^*$	$\ln r_{b(t-1)}$	$\ln \frac{r_{b(t)}}{r_{b(t-1)}}^*$	$\ln r_{k(t-2)}^*$	$\ln r_{L(t-3)}^*$	$\ln \frac{W}{P_Y}_t$
Coefficients:	-7.000 (-2.979)	-.047 (-.405)	-.015 (-.330)	-.240 (-1.029)	-.225 (-4.607)	-.201 (-2.409)	.095 (.343)
Weights:							
w(0)	.147 (1.586)	1.700 (.432)		.413 (.968)	.071 (.628)	.443 (1.275)	
w(1)	.381 (4.697)	.473 (.462)		-.160 (-.319)	.138 (1.605)	.719 (1.952)	
w(2)	.327 (7.066)	-.277 (-.299)		.193 (.905)	.364 (6.406)	.178 (1.026)	
w(3)	.146 (1.765)	-.639 (-.359)		.553 (1.612)	.426 (4.293)	-.341 (-.934)	
w(4)	.000	-.700 (-.361)		.000	.000	.000	
w(5)		-.545 (-.344)					

w(6)	-.264
	(-.284)
w(7)	.058
	(.189)
w(8)	.332
	(.515)
w(9)	.472
	(.475)
w(10)	.390
	(.458)
w(11)	.000

[a]Figures in parentheses are t-statistics, the intercept is 2.009, $\bar{R}^2 = .575$, Durbin-Watson = 1.70, $\rho = .474$, and $\rho' = .254$.

Table 3.8. Generalized-least-squares regression results for the demand for real loans,[a] 1949-I through 1969-IV

Dependent variable: $\ln L/P_Y$

Independent variables:	$\ln \frac{P_{Y(t)}}{P_{Y(t-1)}}^*$	$\ln r_t^*$	$\ln r_{b(t-1)}$	$\ln \frac{r_{b(t)}}{r_{b(t-1)}}^*$	$\ln r_{k(t-2)}^*$	$\ln r_{L(t-3)}^*$	$\ln \frac{W}{P_Y}\ t$
Coefficients:	2.416 (1.525)	.687 (5.812)	-.012 (-.430)	-.008 (-.052)	.064 (1.466)	-.434 (-.585)	1.827 (7.497)
Weights:							
w(0)	.261 (1.935)	.070 (1.527)		.503 (.056)	.430 (1.727)	-.366 (-.251)	
w(1)	.660 (2.609)	.069 (2.700)		-.925 (-.038)	.157 (.925)	.861 (.688)	
w(2)	.269 (3.993)	.076 (2.937)		.149 (.033)	.185 (1.483)	.583 (.801)	
w(3)	-.190 (-.686)	.086 (3.185)		1.274 (.063)	.228 (1.296)	-.078 (-.083)	
w(4)	.000	.099 (4.179)		.000	.000	.000	
w(5)		.110 (6.366)					

w(6)	.118 (8.275)
w(7)	.119 (6.309)
w(8)	.110 (4.475)
w(9)	.089 (3.466)
w(10)	.054 (2.877)
w(11)	.000

[a]Figures in parentheses are t-statistics, the intercept is -1.359, $\overline{R}^2$ = .968, Durbin-Watson = 1.64, ρ = .711, and ρ' = .479.

in these two equations. Namely, all other significant coefficients remained in the same order of magnitude, while the elasticity coefficient for real wealth increased, from 1.486 to 2.029 and from 1.343 to 1.827 respectively. All of the significant coefficients in the real demand for bonds also remained in the same order of magnitude except that the elasticity coefficient for real wealth fell from .466 to .094.

It is interesting to note that if one takes the sum of the elasticity coefficients of real income and real wealth in the portfolio with income, these sums appear in the same order of magnitude as the elasticity coefficients of real wealth in the portfolio without income. Considering the changes created by the omission of real income, the evidence seems to strongly suggest that real wealth is properly the sole constraint upon the portfolio.

The Demand for Real Money Balances

Using the same statistical procedures as described above, the real demand for money was estimated without attempting to choose the lag structures from the portfolio overview. In other words, the lag structure for each independent variable was chosen upon the basis of the standard error of the entire equation. While attempting to minimize the standard error of the entire equation, Tobin's criteria of the same "variables" appearing in all equations was thus violated and the expected returns formulated on the basis of a single equation. Staying within the framework already presented, two estimations of the real demand for money are reported, i.e., with and without real income.

Table 3.9. Generalized-least-squares regression results for the demand for real money,[a] 1949-I through 1969-IV

Dependent variable: $\ln M^s/P_Y$

Independent variables:	$\ln Y_t$	$\ln \frac{P_Y(t)}{P_Y(t-1)}^*$	$\ln r_t^*$	$\ln r_{b(t-1)}$	$\ln \frac{r_b(t)}{r_b(t-1)}^*$	$\ln r^*_{k(t-2)}$	$\ln r^*_{L(t-3)}$	$\ln \frac{W}{P_Y}{}_t$
Coefficients:	.238	-1.204	-.124	-.040	.087	.142	-.093	.453
	(3.710)	(-1.219)	(-2.663)	(-2.860)	(1.200)	(5.438)	(-2.880)	(3.652)
Weights:								
w(0)		.277	-.046		-.114	.213	.247	
		(1.662)	(-.350)		(-.511)	(4.076)	(1.303)	
w(1)		.315	.149		.674	.214	.466	
		(2.695)	(1.879)		(2.242)	(5.562)	(2.741)	
w(2)		.247	.240		.457	.200	.277	
		(4.133)	(3.002)		(4.107)	(7.051)	(2.921)	
w(3)		.132	.252		-.017	.172	.011	
		(1.193)	(4.375)		(-.075)	(8.242)	(.060)	
w(4)		.029	.209		.000	.129	.000	
		(.212)	(3.895)			(3.524)		
w(5)		.000	.137			.072		
			(1.725)			(1.866)		
w(6)			.059			.000		
			(.783)					
w(7)			.000					

[a]Figures in parentheses are t-statistics, the intercept is .045, $\bar{R}^2$ = .700, Durbin-Watson = 1.687, ρ = .716, and ρ' = .463.

The results obtained by estimating the real demand for money with real income as an independent variable are presented in Table 3.9. In order to estimate the real demand for money "in isolation," it was deemed necessary to alter the lag structures of three independent variables. These were: (1) the change in output prices was increased from a 4- to a 5-period lag; (2) the rate of return to time deposits was decreased from 11- to a 7-period lag; and (3) the rate of return to capital was increased from a 4- to a 6-period lag. All other variables retained their previous structure and all variables maintained any discrete lags stated above.

Compared with the real demand for money with real income as an independent variable previously reported, the rate of return to capital still appears as the only independent variable which possesses an incorrect a priori sign. However, only five of these seven variables exhibit statistical significance at the .05 level. Both the change in output prices and the change in the rate of return to bonds are no longer significant. The argument in favor of a speculative motive for holding money is lost. No other significant differences are noticed although the overall interest-elasticity, as computed by summing the elasticity coefficients upon the rate of return to time deposits, bonds, and loans, does increase slightly, i.e., from -.223 to -.257.

The results obtained by estimating the real demand for money omitting real income as an independent variable are presented in Table 3.10. Compared with the real demand for money without real income previously reported, the rate of return to capital still appears

Table 3.10. Generalized-least-squares regression results for the demand for real money,[a] 1949-I through 1969-IV

Dependent variable: ln M^s/P_γ

Independent variables:	ln $\frac{P_\gamma(t)}{P_\gamma(t-1)}^*$	ln r^*_t	ln $r_{b(t-1)}$	ln $\frac{r_b(t)}{r_b(t-1)}^*$	ln $r^*_{k(t-2)}$	ln $r^*_{L(t-3)}$	ln $\frac{W}{P_\gamma}_t$
Coefficients:	-1.143	-.067	-.033	.108	.156	-.117	.756
	(-1.034)	(-1.375	(-2.121)	(1.325)	(5.534)	(-3.324)	(7.277)
Weights:							
w(0)	.286	-.235		.031	.191	.281	
	(1.410)	(-.665)		(.227)	(3.724)	(1.671)	
w(1)	.283	.148		.560	.184	.344	
	(2.297)	(.896)		(2.893)	(5.006)	(2.527)	
w(2)	.227	.320		.384	.185	.260	
	(2.991)	(1.653)		(5.557)	(6.495)	(3.089)	
w(3)	.144	.338		.024	.181	.116	
	(1.181)	(2.138)		(.133)	(8.779)	(.754)	
w(4)	.060	.258		.000	.157	.000	
	(.412)	(2.062)			(4.517)		
w(5)	.000	.137			.102		
		(.831)			(2.759)		
w(6)		.033			.000		
		(.203)					
w(7)		.000					

[a]Figures in parentheses are t-statistics, the intercept is -.077, $\bar{R}^2$ = .649, Durbin-Watson = 1.603, ρ = .663, and ρ' = .516.

as the only independent variable which possesses an incorrect a priori sign. However, only three of these six variables are statistically significant at the .05 level. Again, the change in the bond rate is no longer significant. Also, as exhibited in the previous results, the elasticity coefficient for real wealth increased from .453 to .756. This change represents approximately the summation of the elasticity coefficients for real income and real wealth in the previous single equation formulation.

These results provide some interesting comparisons of using different specifications for estimation of the demand for financial assets. In particular, these results demonstrate rather clearly that a Smith-Tobin framework illuminates many pitfalls of financial model-building.

CHAPTER IV. SUMMARY AND CONCLUSIONS

For the past three decades, several economists have engaged in theoretical and empirical attempts to clarify the nature of the demand for money and the determination of the relative prices of financial assets. While most of these attempts recognize the importance of wealth as a constraint, the definition of wealth appropriate to such analyses has often not been clearly delineated. This lack of clarity concerning the constraint upon the portfolio of various assets, has resulted in a lack of specification of the precise assets among which the wealth is to be allocated. Therefore, the majority of evidence presented in behalf of various specifications of the demand for money consists of formulations of the demand for this single asset recognized to be only one of the assets within the portfolio. The recognition of the other assets within the portfolio has generally been provided by considering a single rate of return as representative of a vector of rates associated with the entire array of assets appropriate to the portfolio. This study has reported evidence which purports to magnify the importance of such a simplification.

Two specific conclusions can be drawn from this study: (1) aggregate portfolio adjustments occur; and (2) specifications of the demand for money within the specified portfolio context provide additional information concerning the nature of the demand for money. Considering the criteria presented in this study for judging the success of an estimation of the demands for assets within a portfolio, it can be concluded that the attempt to estimate such demand functions was successful. Also, the information provided by these estimations strongly suggest that in the aggregate the private sector does indeed allocate its wealth among a portfolio of assets.

Specification of the demand for money within the context of the portfolio of net non-human wealth of the private sector resulted in many interesting observations concerning the nature of the demand for money. First, it should be noted that all of the evidence supports the theory of an interest-elastic demand for money, i.e., the demand for money is responsive to the level of interest. The frequency with which this proposition has been demonstrated not only with regard to the four estimations presented in this study but also, with few exceptions in previous empirical research, would seem to leave little doubt concerning this behavioral hypothesis.

Secondly, attempts to approximate the rate of return to the array of assets with a single rate may be a more serious oversimplification than often thought. When an overall view of the demands for the assets within the portfolio is adopted, it is easily seen that the entire array of perceived asset returns appear significant only in the demand for money. It is of interest to note, that compared to other assets within the portfolio, money is the only asset which provides its holder only with "income-in-kind." Possibly opportunity costs become relatively more important if an asset has no immediately visable return.

In any case, the specification of the demand for money within a specific portfolio provides strong evidence for a speculative motive for holding money as well as some evidence for an extropolative expectations formulation hypothesis. In general, no such evidence was forthcoming from a single equation formulation. Finally, income was found to provide no additional information when wealth was included as an explanatory variable in the demands for financial assets. It would appear that income becomes a redundant concept when both rates of return and wealth are included within estimations of portfolio models.

This study would suggest several areas for continued research: (1) the optimization process of portfolio adjustment; (2) disaggregated portfolio models; (3) expectations formulation; and (4) portfolio estimations of demands for assets within various sectors of the economy. The model and procedure utilized in this paper should provide a framework to resolve these issues.

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APPENDIX A. PREVIOUS EMPIRICAL RESULTS

I. Results presented by Latane′ (1954):

1. $\frac{M}{Y} = \frac{.00795304}{r} + .100418$ (OLS)

$R^2 = .911$

2. $\frac{1}{r} = 111.775 \frac{M}{Y} - 7.233$ (OLS)

Equations were fit utilizing data from 1919-52 excluding 1932, 1933, 1942, 1946, and 1947. The series used were defined as follows:

M: Is demand deposits adjusted plus currency in circulation on mid-year call date.

Y: Is the gross national product.

r: Is the interest rate on high-grade long-term corporate obligations.

II. Results presented by Brunner and Meltzer (1964):

			r	W/P_a	Y/Y_p	P_y	σ_1	R^2	D-W
1.	linear regression	TSLS	-14.609	.249		.164		.989	.77
			(-9.48)	(5.80)		(1.06)			
	elasticity at mean	TSLS	-.577	1.813		.266			
		OSLS	-.526	1.711		.354			.73
2.	linear regression	TSLS	-18.994	.201	-54.722	.347		.992	1.76
			(-10.861)	(5.14)	(-4.05)	(2.73)			
	elasticity at mean	TSLS	-.750	1.463	-.772	.644			
		OSLS	-.652	1.420	-.597	.661			1.57

			r	W/P_a	Y/Y_p	P_y	σ_1	R^2	D-W
3.	linear regression		-20.064	.186	-62.198	.464	9.228	.992	1.80
			(-8.54)	(4.31)	-(3.66)	(2.81)	(.97)		
	elasticity at mean	TSLS	-.793	1.354	-.876	.753	.02		
		OSLS	-.652	1.420	-.597	.661	-0		1.57
4.	linear regression		-18.296	.190	-52.306	.436	4.632	.992	1.73
			(-8.55)	(4.64)	(-3.32)	(2.78)	(.521)		
	elasticity at mean	TSLS	-.723	1.383	-.738	.708	.011		
		OSLS	-.652	1.420	-.597	.661	-0		1.57

Figures in parentheses are t-statistics, the dependent variable is M_1. Estimates of the equations were derived from OSLS and TSLS for linear approximation of several specifications of the demand for money. The data utilized was from 1930-1959 and the series used were defined as follows:

M_1: Is demand deposits adjusted plus currency outside banks.

r : Is the bond yield (long-term rate on U. S. Government securities).

$\frac{W}{P_a}$: Is an adjusted Goldsmith measure of the public's tangible and non-human deflated according to an appropriate price index.

$\frac{Y}{Y_p}$: Is the ratio of current net income to Friedman's permanent income.

P_Y: Is a deflator of net national product.

σ_1: Is a moving three-year standard deviation of the bond yield.

III. Results presented by Hamburger (1966):

1. $\bar{M}_1^* = -.0039 - \underset{(4.13)}{.270}\ \bar{\rho}_B^* + \underset{(1.22)}{.891}\ \bar{Y}'^*$

$\bar{R}^2 = .285$, and DW = .87.

2. $\bar{M}_1^* = -.0006 - .160\ \bar{\rho}_B^* - .130\ \bar{\rho}_E^* - .018\ \bar{Y}'^*$
(2.29) (3.16) (.117)

$\bar{R}^2 = .427$, and DW = 1.74.

3. $\bar{M}_1^* = .0004 - .161\ \bar{\rho}_B^* - .165\ \bar{\rho}_E^* - .136\ W_1^*$
(2.30) (2.62) (.88)

$\bar{R}^2 = .436$, and DW = 1.84.

4. $\bar{M}_1^* = -.273 - .185\ \bar{\rho}_B^* - .124\ \bar{\rho}_E^* + .253\ W_2^*$
(2.64) (2.95) (1.29)

$\bar{R}^2 = .432$, and DW = 1.91.

Figures in parentheses are t-statistics. Estimates of the equations as first-difference relationships were derived from OLS. The data utilized was from 1952-1960 and the series used were defined as follows:

M_1: Is the sum of currency plus demand deposits.

ρ_B: Is Moody's Aaa rate on long-term corporate bonds.

ρ_E: Is Moody's dividend yield.

Y': Is disposable personal income measured in current prices.

Y : Is disposable personal income measured in real terms.

W_1: Is a measure of all asset components in dollars of current market value.

W_2: Is the ratio of disposable personal income to personal consumption expenditures.

The bar (—) indicates relative first difference and the asterisk (*) indicates weighted average.

APPENDIX B. FURTHER EMPIRICAL RESULTS

Table B.1. Generalized-least-squares regression results for the demand for real money,[a] 1949-I through 1969-IV

Dependent variable: $\ln M^s/P_Y$

Independent variables:	$\ln Y^*$	$\ln r_t^*$	$\ln r_b^*$	$\ln \frac{r_b(t)}{r_b(t-1)}^*$	$\ln r_k^*$	$\ln \frac{(P_k/P_Y)(t)}{(P_k/P_Y)(t-1)}^*$	$\ln r_L^*$	$\ln \frac{W}{P_Y}$
Coefficients:	.172 (1.723)	-.080 (-1.785)	-.104 (-3.301)	.382 (2.739)	.130 (5.837)	.169 (1.405)	-.006 (-.153)	.458 (3.154)
Weights:								
w(0)	.820 (1.469)	.186 (1.162)	.478 (2.571)	.174 (3.378)	.101 (1.415)	.148 (1.678)	-3.781 (-.145)	
w(1)	.455 (1.281)	.386 (1.867)	.421 (4.731)	.334 (8.589)	.230 (5.161)	.294 (2.600)	-3.666 (-.145)	
w(2)	.099 (.369)	.454 (1.783)	.219 (3.404)	.298 (14.602)	.275 (9.358)	.302 (3.204)	-1.974 (-.137)	
w(3)	-.155 (-.517)	.419 (1.776)	-.002 (-.017)	.164 (4.753)	.245 (5.823)	.221 (5.915)	.433 (.187)	
w(4)	-.219 (-.569)	.311 (1.886)	-.117 (-1.036)	.031 (.701)	.150 (2.909)	.101 (1.637)	2.696 (.161)	
w(5)	.000	.160 (2.243)	.000	.000	.000	-.008 (-.073)	3.952 (.156)	

w(6)	-.005 (-.065)	-.058 (-.554)	3.340 (.154)
w(7)	-.155 (-.865)	.000	.000
w(8)	-.258 (-1.053)		
w(9)	-.287 (-1.130)		
w(10)	-.211 (-1.171)		
w(11)	.000		

[a]Figures in parentheses are t-statistics, the intercept is .147, $\bar{R}^2$ = .874, Durbin-Watson = 1.584, ρ = .552, and ρ' = .471.

Table B.2. Generalized-least-squares regression results for the demand for real time deposits,[a] 1949-I through 1969-IV

Dependent variable: $\ln T^s/P_Y$

Independent variables:	$\ln Y^*$	$\ln r_t^*$	$\ln r_b^*$	$\ln \frac{r_{b(t)}}{r_{b(t-1)}}^*$	$\ln r_k^*$	$\ln \frac{(P_k/P_Y)_{(t)}}{(P_k/P_Y)_{(t-1)}}^*$	$\ln r_L^*$	$\ln \frac{W}{P_Y}$
Coefficients:	1.622 (6.715)	.644 (6.090)	-.202 (-2.657)	.106 (.308)	-.116 (-2.172)	-.180 (-.622)	-.264 (-3.035)	.505 (1.416)
Weights:								
w(0)	.207 (1.773)	.141 (3.009)	.443 (1.922)	.321 (.495)	.196 (.969)	-.031 (-.112)	.271 (1.322)	
w(1)	.256 (2.984)	.114 (3.971)	.363 (3.546)	.360 (.935)	.262 (2.138)	-.046 (-.132)	.253 (2.208)	
w(2)	.245 (3.546)	.097 (3.253)	.204 (2.439)	.252 (1.339)	.253 (3.000)	.038 (.145)	.209 (1.848)	
w(3)	.189 (3.303)	.090 (2.946)	.041 (.335)	.091 (.219)	.191 (1.641)	.167 (2.050)	.149 (1.728)	
w(4)	.102 (1.205)	.089 (3.460)	-.052 (-.400)	-.025 (-.051)	.099 (.701)	.284 (1.399)	.086 (1.041)	
w(5)	.000	.091 (5.054)	.000	.000	.000	.332 (.944)	.032 (.282)	

w(6)	.092 (6.069)	.256 (.800)	-.000 (-.003)
w(7)	.091 (4.192)	.000	.000
w(8)	.084 (2.932)		
w(9)	.069 (2.292)		
w(10)	.042 (1.931)		
w(11)	.000		

[a]Figures in parentheses are t-statistics, the intercept is -2.534, $\bar{\bar{R}}^2$ = .993, Durbin-Watson = 1.352, ρ = .487, and ρ' = .463.

Table B.3. Generalized-least-squares regression results for the demand for real bonds,[a] 1949-I through 1969-IV

Dependent variable: $\ln B_p^s/P_Y$

Independent variables:	$\ln Y^*$	$\ln r_t^*$	$\ln r_b^*$	$\ln \frac{r_b(t)}{r_b(t-1)}^*$	$\ln r_k^*$	$\ln \frac{(P_k/P_Y)(t)}{(P_k/P_Y)(t-1)}^*$	$\ln r_L^*$	$\ln \frac{W}{P_Y}$
Coefficients:	-.326 (-.824)	.155 (1.134)	.167 (1.328)	-2.008 (-3.047)	-.081 (-1.012)	.556 (1.171)	-.153 (-1.288)	-.042 (-.065)
Weights:								
w(0)	-.100 (-.071)	.189 (.506)	.878 (1.130)	.225 (3.596)	.595 (.709)	-.047 (-.225)	.648 (.829)	
w(1)	-.711 (-.527)	.349 (1.165)	.585 (1.602)	.285 (7.605)	.097 (.225)	.057 (.461)	.701 (1.386)	
w(2)	-.116 (-.140)	.401 (1.009)	.141 (.556)	.253 (10.114)	.010 (.025)	.148 (1.914)	.495 (1.133)	
w(3)	.795 (.855)	.368 (.915)	-.244 (-.552)	.168 (4.597)	.113 (.296)	.216 (3.356)	.156 (.511)	
w(4)	1.131 (.802)	.275 (.880)	-.361 (-.764)	.070 (1.599)	.184 (.410)	.246 (2.540)	-.190 (-.540)	
w(5)	.000	.146 (.844)	.000	.000	.000	.229 (1.852)	-.416 (-.821)	

w(6)	.006	.151	-.395
	(.049)	(1.476)	(-.862)
w(7)	-.120	.000	.000
	(-.455)		
w(8)	-.208		
	(-.554)		
w(9)	-.234		
	(-.587)		
w(10)	-.173		
	(-.600)		
w(11)	.000		

[a]Figures in parentheses are t-statistics, the intercept is 4.334, $\bar{R}^2$ = .623, Durbin-Watson = 1.582, ρ = .240, and ρ' = .217.

Table B.4. Generalized-least-squares regression results for the demand for real loans,[a] 1949-I through 1969-IV

Dependent variable: $\ln L^s/P_Y$

Independent variables:	$\ln Y^*$	$\ln r_t^*$	$\ln r_b^*$	$\ln \frac{r_{b(t)}}{r_{b(t-1)}}^*$	$\ln r_k^*$	$\ln \frac{(P_k/P_Y)_{(t)}}{(P_k/P_Y)_{(t-1)}}^*$	$\ln r_L^*$	$\ln \frac{W}{P_Y}$
Coefficients:	1.840	.183	-.077	-.094	-.124	.062	-.084	-.079
	(12.331)	(2.688)	(-1.658)	(-.454)	(-3.727)	(.344)	(-1.520)	(-.363)
Weights:								
w(0)	.140	-.048	-.105	.314	.156	.041	.506	
	(2.270)	(-.412)	(-.282)	(.735)	(1.392)	(.113)	(1.126)	
w(1)	.247	.066	.197	.369	.272	.116	.046	
	(5.439)	(1.111)	(1.309)	(1.359)	(4.036)	(.455)	(.192)	
w(2)	.267	.137	.340	.259	.276	.170	-.100	
	(7.313)	(1.883)	(2.459)	(2.162)	(5.964)	(.855)	(-.344)	
w(3)	.221	.173	.344	.090	.199	.199	-.042	
	(7.299)	(2.124)	(1.803)	(.317)	(3.062)	(1.557)	(-.191)	
w(4)	.125	.180	.224	-.032	.092	.200	.109	
	(2.799)	(2.545)	(1.172)	(-.095)	(1.168)	(1.142)	(.696)	
w(5)	.000	.166	.000	.000	.000	.170	.240	
		(3.441)				(.666)	(1.012)	

w(6)	.137 (4.194)	.104 (.460)	.241 (1.006)
w(7)	.099 (2.107)	.000	.000
w(8)	.060 (.889)		
w(9)	.026 (.352)		
w(10)	.004 (.067)		
w(11)	.000		

[a]Figures in parentheses are t-statistics, the intercept is -1.396, $\bar{R}^2$ = .996, Durbin-Watson = 1.878, ρ = .604, and ρ' = .381.

APPENDIX C. ALMON LAG TECHNIQUE

In most studies, incorporation of a distributed-lag function merely states the independent variable as a weighted sum of the past values of the independent variable. Estimation of such a distributed-lag function has, in general, relied upon the assumption of a geometrically decaying weight structure. In 1965, Shirley Almon introduced a technique for estimating distributed-lag weight structures which does not require the a priori assumption of a particular shape for the weight structure, but instead requires that the weight structure be approximated well by a polynomial of some specified degree.

In particular, the problem is to fit equations of the following type:

$$(1) \quad \ln Y_t^s = \ln \alpha_0 + \sum_{i=0}^{m_1} \alpha_1 w_1(i) \ln X_{1,t-i} + \sum_{i=0}^{m_2} \alpha_2 w_2(i) \ln X_{2,t-i} + \dots + \sum_{i=0}^{m_n} \alpha_n w_n(i) \ln X_{n,t-i} + \mu_t$$

where the Y's and X's are observable and the α's and w(i)'s must be simultaneously estimated. This may be accomplished by using the Lagrange polynomial interpolation method to estimate the distributed-lag weight structures. Thus, it is assumed that the $w_j(i)$'s are values of polynomials of degree q in i ($q < m_j$), i.e.,

$$(2) \quad w_j(i) = \sum_{h=0}^{q} {}_j a_h \, i^h \qquad i=0, 1, \dots, m_j; \quad j=1, 2, \dots, n.$$

Multiplying both sides of equation 2 by α_j, the following result is obtained:

$$(3) \quad \alpha_j w_j(i) = \sum_{h=0}^{q} \alpha_j \, {}_ja_h \, i^h \qquad i=0, 1, \ldots, m_j; \; j=1, 2, \ldots, n.$$

Therefore, the coefficients of the $\ln X_{j,t-i}$ [i.e., $\alpha_j w_j(i)$] are shown to be values of polynomials of degree q in i.

At this point, suppose that the values of $\alpha_j w_j(i_k) = {}_jb_k$ are known for q+1 values of i. In other words, it is assumed that $\alpha_j w_j(i_1) = {}_jb_1$, $\alpha_j w_j(i_2) = {}_jb_2$, ..., $\alpha_j w_j(i_q) = {}_jb_q$, $\alpha_j w_j(i_{q+1}) = {}_jb_{q+1}$. Then the method of Lagrange may be employed in order to calculate all the $\alpha_j w_j(i)$'s as linear combinations of these known values as follows:

$$(4) \quad \alpha_j w_j(i) = \sum_{k=1}^{q+1} {}_j\Psi_k(i) \, {}_jb_k \qquad i=0, 1, \ldots, m_j; \; j=1, 2, \ldots, n,$$

where the ${}_j\Psi_k(i)$'s are values of the Lagrangian polynomial coefficients,

$$(5) \quad {}_j\Psi_k(i) = \frac{\prod_{\substack{s=1 \\ s\neq k}}^{q+1} (i-i_s)}{\prod_{\substack{s=1 \\ s\neq k}}^{q+1} (i_k-i_s)} \qquad i=0, 1, \ldots, m_j; \; k=1, 2, \ldots, q+1; \; j=1, 2, \ldots, n.$$

It is assumed throughout that the last period's weight is known to be zero, i.e., $w_j(m_j) = 0$; hence, always using the last period as the q+1 known point, $\alpha_j w_j(m_j) = {}_jb_{m_j} = {}_jb_{q+1} = 0$, equation 4 reduces to the following:

$$(6) \quad \alpha_j w_j(i) = \sum_{k=1}^{q} {}_j\Psi_k(i) \, {}_jb_k \qquad i=0, 1, \ldots, m_j; \; j=1, 2, \ldots, n.$$

Substituting equation 6 into 1, the following results are obtained:

$$(7) \quad \ln Y_t^s = \ln \alpha_0 + \sum_{i=0}^{m_1} \left[\sum_{k=1}^{q} {}_1\Psi_k(i) \; {}_1b_k \right] \ln X_{1,t-i}$$

$$+ \sum_{i=0}^{m_2} \left[\sum_{k=1}^{q} {}_2\Psi_k(i) \; {}_2b_k \right] \ln X_{2,t-i} + \ldots$$

$$+ \sum_{i=0}^{m_n} \left[\sum_{k=1}^{q} {}_n\Psi_k(i) \; {}_nb_k \right] \ln X_{n,t-i} + \mu_t$$

and then rearranging terms,

$$(8) \quad \ln Y_t^s = \ln \alpha_0 + \sum_{k=1}^{q} {}_1b_k \left[\sum_{i=0}^{m_1} {}_1\Psi_k(i) \ln X_{1,t-i} \right]$$

$$+ \sum_{k=1}^{q} {}_2b_k \left[\sum_{i=0}^{m_2} {}_2\Psi_k(i) \ln X_{2,t-i} \right] + \ldots$$

$$+ \sum_{k=1}^{q} {}_nb_k \left[\sum_{i=0}^{m_n} {}_n\Psi_k(i) \ln X_{n,t-i} \right] + \mu_t$$

Let

$$(9) \quad {}_j^kA_t = \sum_{i=0}^{m_j} {}_j\Psi_k(i) \ln X_{j,t-i} \qquad k=1, 2, \ldots, q$$

$$j=1, 2, \ldots, n$$

and substitute into equation 8, i.e.,

$$(10) \quad \ln Y_t^s = \ln \alpha_0 + \sum_{k=1}^{q} {}_1b_k \; {}_1^kA_t$$

$$+ \sum_{k=1}^{q} {}_2b_k \; {}_2^kA_t$$

$$+ \ldots + \sum_{k=1}^{q} {}_nb_k \; {}_n^kA_t + \mu_t.$$

Given values for the lag lengths, m_j, and for the degree of the polynomial, q, then the ${}_j^kA$ (Almon variables) can be calculated from empirical observations of X_j according to equation 9. Equation 10 can be fitted by means of least squares to yield estimates of ln α_0 and the ${}_jb_k$. Estimates of the $\alpha_j w_j(i)$ may be obtained once estimates of the ${}_jb_k$ are found and substituted into equation 6. Also, since the sum of the weights are assumed to be one, i.e., $\sum_{i=0}^{m_j} w_j(i) = 1$, then $\sum_{i=0}^{m_j} \alpha_j w_j(i) = \alpha_j$, j=1, 2, ..., n and it is relatively easy to calculate estimates of the α_j.

The

$$(11) \quad \text{VAR}\,[\alpha_j w_j(i)] = \sum_{k=1}^{q} [{}_j\Psi_k(i)]^2 \ \text{VAR}\,({}_jb_k)$$

and

$$(12) \quad \text{VAR}\,(\alpha_j) = \sum_{k=1}^{q} \left[\sum_{i=0}^{m_j} {}_j\Psi_k(i)\right] \text{VAR}\,({}_jb_k),$$

where VAR $({}_jb_k)$ is obtained in the usual manner. Computation of VAR $[w_j(i)]$ is troublesome since the ratio of two normally distributed variates is not normally distributed. Therefore, the following formula was used as an approximation:

$$(13) \quad \text{VAR}\,[w_j(i)] = \frac{\text{VAR}[\alpha_j w_j(i)] - 2w_j(i)\ \text{COV}[\alpha_j w_j(i), \alpha_j] + w_j(i)^2\ \text{VAR}\,(\alpha_j)}{\alpha_j^{\,2}}.$$

APPENDIX D: PORTFOLIO DEMAND FUNCTIONS

The following assumptions are made in order to investigate an individual's real demand for any asset other than money within the portfolio: (1) The individual is a "price-taker," i.e., the real rates of return to the various assets whether market-determined or institutionally-determined cannot be influenced by the actions of a single individual. (A logical consequence of this assumption is that the individual is a "risk-taker," i.e., risk associated with a particular asset as measured by the standard deviation of the price of the asset does not alter with the proportion of a particular asset held by an individual. That is, if an individual cannot influence the price of an asset, then an individual cannot influence price changes. This does not imply that an individual does not influence the total risk involved when choosing a particular portfolio of assets. Indeed, it will be argued that an individual seeks to manipulate both the total risk as well as the total income received from his portfolio.) (2) Risk associated with a particular asset remains constant in equilibrium.

While the second assumption must hold for all assets, money was exempted from the list due to the first assumption. Since the real return to money is received only "in kind," the decision of an individual to hold more real balances will be accompanied by a lower real rate of return to money and visa versa. Also, to the extent that the real return to an individual's real holdings of time deposits is influenced by income "in kind," the decision of an individual to hold more of these deposits will be accompanied by a lower real rate of return to this asset.

Recalling the above discussion, consider an individual whose utility function includes two arguments. These are: (1) real income derived from the portfolio and (2) the income risk associated with the portfolio.

$$(1) \quad U = U(TR, Z) \qquad \frac{\partial U}{\partial TR} > 0; \ \frac{\partial U}{\partial Z} < 0$$

where TR ≡ total real income derived from the portfolio

Z ≡ total income risk associated with the portfolio

The total real income derived from the portfolio depends upon the volume of each asset held and the real rate of return associated with each asset.

$$(2) \quad TR = TR\,[\,f(M,\gamma),\ T,\ RR_T,\ B_pP_b,\ KP_K,\ RR_K,\ L,\ RR_L]$$

$$f_M(M,\gamma) > 0,\ f_\gamma(M,\gamma) > 0, \frac{\partial\, TR}{\partial\, T} > 0,\ \frac{\partial TR}{\partial RR_T} > 0,\ \frac{\partial TR}{\partial B_pP_b} > 0,$$

$$\frac{\partial TR}{\partial RR_B} > 0,\ \frac{\partial TR}{\partial KP_K} > 0,\ \frac{\partial TR}{\partial RR_K} > 0,\ \frac{\partial TR}{\partial L} < 0,\ \frac{\partial TR}{\partial RR_L} < 0$$

where $f(M,\gamma)$ = contribution to the total income due to real money balances. In accordance with the above, $f_{MM}(M,\gamma) > 0$ and $f_M\ (M,\gamma) < 0$.

The total risk associated with a particular portfolio depends upon the proportion of each asset held.

$$(3) \quad Z = Z\left(\frac{M}{W}, \frac{T}{W}, \frac{\bar{B}_p\bar{P}_b}{W}, \frac{\overline{KP}_K}{W}, \frac{L}{W}\right)$$

$$\frac{\partial Z}{\partial \frac{M}{W}} > 0,\ \frac{\partial Z}{\partial \frac{T}{I}} > 0,\ \frac{\partial Z}{\partial \frac{B_pP_b}{W}} > 0,\ \frac{\partial Z}{\partial \frac{KP_K}{W}} > 0,\ \frac{\partial Z}{\partial \frac{L}{W}} > 0$$

where the holdings of each asset are expressed as a proportion of the total wealth.

Portfolio selection can be viewed as the process of maximizing utility subject to a wealth constraint.

(4) $W = M + T + BP_b + KP_K - L$

where the net non-human wealth is viewed as the individual's constraint upon utility maximizations. In order to find the constrained maximum, the method of Lagrange undetermined multipliers will be utilized. From equations 1, 2, 3, and 4, form the following problem:

Maximize $U = U(TR,Z)$

Subject to $W = M + T + B_pP_b + KP_K - L.$

Applying the method of Lagrange's multiplier gives

(i) $$\frac{\partial\Psi}{\partial M} = \frac{\partial U}{\partial TR} \cdot f_M(M,\gamma) + \frac{\partial U}{\partial Z}\frac{\partial Z}{\partial \frac{M}{W}}\frac{1}{W} + \lambda = 0$$

(ii) $$\frac{\partial\Psi}{\partial T} = \frac{\partial U}{\partial TR}\frac{\partial TR}{\partial T} + \frac{\partial U}{\partial Z}\frac{\partial Z}{\partial \frac{T}{W}}\frac{1}{W} + \lambda = 0$$

(iii) $$\frac{\partial\Psi}{\partial B_pP_b} = \frac{\partial U}{\partial TR}\frac{\partial TR}{\partial B_pP_b} + \frac{\partial U}{\partial Z}\frac{\partial Z}{\partial \frac{B_pP_b}{W}}\frac{1}{W} + \lambda = 0$$

(iv) $$\frac{\partial\Psi}{\partial KP_K} = \frac{\partial U}{\partial TR}\frac{\partial TR}{\partial KP_K} + \frac{\partial U}{\partial Z}\frac{\partial Z}{\partial \frac{KP_K}{W}}\frac{1}{W} + \lambda = 0$$

(v) $\frac{\partial \Psi}{\partial L} = \frac{\partial U}{\partial TR}\frac{\partial TR}{\partial L} + \frac{\partial U}{\partial Z}\frac{\partial Z}{\partial \frac{L}{W}}\frac{1}{L} + \lambda = 0$

(vi) $\frac{\partial \Psi}{\partial \lambda} = W - M - T - B_p P_b - KP_K + L = 0$

where $\Psi = U(TR,Z) + \lambda(W-M-T-B_p P_b - KP_K + L)$.

Solving equations (i) through (v) pairwise for $-\lambda$ and dividing, results in the following:

$$\text{(vii)}\quad \frac{\frac{\partial U}{\partial TR} \cdot f_M(M,\gamma) + \frac{\partial U}{\partial Z}\frac{\partial Z}{\partial \frac{M}{W}}\frac{1}{W}}{\frac{\partial U}{\partial TR}\frac{\partial TR}{\partial T} + \frac{\partial U}{\partial Z}\frac{\partial Z}{\partial \frac{T}{W}}\frac{1}{W}} = 1$$

$$\text{(viii)}\quad \frac{\frac{\partial U}{\partial TR}\frac{\partial TR}{\partial T} + \frac{\partial U}{\partial Z}\frac{\partial Z}{\partial \frac{T}{W}}\frac{1}{W}}{\frac{\partial U}{\partial TR}\frac{\partial TR}{\partial BP_b} + \frac{\partial U}{\partial Z}\frac{\partial Z}{\partial \frac{BP_b}{W}}\frac{1}{W}} = 1$$

(ix) etc.

The ten equations resulting from the pairwise solutions are the familiar marginal conditions for utility maximization, i.e., the marginal utility of the last dollar invested in each asset must be equal. More specifically, equation (vi) states that the marginal utility of income derived from holding money plus the marginal disutility of risk associated with holding money must be in a constant proportion to the marginal utility of income derived from holding time deposits plus the marginal disutility of risk associated with holding time deposits.

Therefore, should the marginal utility of income derived from a particular asset increase due to an increase in the real rate of return to that asset, then the individual will acquire more of that asset until this constant proportional relationship is restored. In terms of the individual's real demand for the various assets, assuming the second order conditions hold, the above conclusion can be stated by noting the sign associated with each real return. That is,

$$1. \quad \frac{M_i^d}{P_Y} = f_1(\overset{(+)}{RR_M}, \overset{(-)}{RR_T}, \overset{(-)}{RR_B}, \overset{(-)}{RR_K}, \overset{(+)}{RR_L}, \overset{(+)}{W})$$

$$2. \quad \frac{T_i^d}{P_Y} = f_2(\overset{(-)}{RR_M}, \overset{(+)}{RR_T}, \overset{(-)}{RR_B}, \overset{(-)}{RR_K}, \overset{(+)}{RR_L}, \overset{(+)}{W})$$

$$3. \quad \frac{(B_p P_b)_i^d}{P_Y} = f_3(\overset{(-)}{RR_M}, \overset{(-)}{RR_T}, \overset{(+)}{RR_B}, \overset{(-)}{RR_K}, \overset{(+)}{RR_L}, \overset{(+)}{W})$$

$$4. \quad \frac{(KP_K)_i^d}{P_Y} = f_4(\overset{(-)}{RR_M}, \overset{(-)}{RR_T}, \overset{(-)}{RR_B}, \overset{(+)}{RR_K}, \overset{(+)}{RR_L}, \overset{(+)}{W})$$

$$5. \quad \frac{L_i^d}{P_Y} = f_5(\overset{(+)}{RR_M}, \overset{(+)}{RR_T}, \overset{(+)}{RR_B}, \overset{(+)}{RR_K}, \overset{(-)}{RR_L}, \overset{(+)}{W})$$

where the subscript i refers to the arbitrary $i^{\underline{th}}$ individual. The aggregate demand functions should possess the same signs since aggregation merely takes place over all individuals, i.e., $\frac{M_d}{P_Y} = \sum_i \frac{M_d^i}{P_Y}$ and assuming no aggregation problem exists. These signs are the a priori signs referred to throughout the text.

APPENDIX E. SOURCES AND DESCRIPTION OF DATA

The following data were taken from the Federal Reserve Bulletin, 1951-1970. The series used were as follows:

M^s: Is a quarterly series of daily averages of the seasonally adjusted money supply. Equals demand deposits at all commercial banks minus cash items in process of collection, minus federal reserve float, plus coins and currency outside of the treasury, federal reserve banks, and the vaults of the commercial banks. This series is in current dollars.

GNP: The "Gross National Product" or expenditures is the market value of the output of goods and services produced by the nation's economy, before deduction of depreciation charges and other allowances for business and institutional consumption of durable goods. This is a quarterly series in current dollars.

T^s: Is a quarterly series of daily averages of the seasonally adjusted time deposits at all commercial banks other than those due to domestic commercial banks and the U. S. Government. This series is in current dollars.

r_k: Is a quarterly series of the end of period earnings/price ratios of Standard and Poor's corporate series.

r_t: Is the maximum rate that may be paid on time and savings deposits by federal reserve member banks as established by the board of governors under provisions of regulation Q.

The following data were taken from the Data Bank Retrieval System, 1971. The series used were as follows:

P_Y: Is the seasonally adjusted quarterly series of the GNP implicit price deflator (1958=100).

r_b: Is a quarterly series of the average of daily closing bid prices on U. S. Government 3 to 5 year securities.

$\frac{r_b\,(t)}{r_b\,(t-1)}$: Is the ratio of a quarterly series of the average daily prices on U. S. Government long term (10 or more years) securities.

r_L: Is a quarterly series of average bank rates on short-term business loans in 35 centers.

P_k: Is a quarterly series of daily averages of the Standard and Poors combined index of common stock prices.

B^s: Is a quarterly series of average end of month privately held marketable interest-bearing U. S. Government securities. This series is in current dollars.

L^s: Is the seasonally adjusted quarterly series of average last Wednesday of the month commercial bank holdings of loans plus commercial bank holdings of securities excluding U. S. Government securities. This series is in current dollars.

The following data were taken from the Federal Reserve-MIT-Penn Econometric Model, 1970. The series used was as follows:

K/P_Y : Is a quarterly series of the end of period sum of stock of consumer durables, net stock of producers' structures, net stock of producers' durables, stock of non-farm business inventory, stock of single family houses, and stock of multi-family houses. This series is in real dollars with each component deflated by its own price index (1958 = 100).